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The Cournot–Bertrand model and the degree of product differentiation
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We consider a duopoly model with a Cournot-type firm and a Bertrand-type firm. Both survive and the equilibrium is stable when there is sufficient product differentiation. With no differentiation, only the Cournot-type firm survives, and the outcome is perfectly competitive.

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1. Introduction

The study of oligopoly began with Cournot (1838) and Bertrand (1883). As is well known, Cournot derived a Nash equilibrium in a duopoly market when firms produce homogeneous goods and compete in output, and Bertrand derived a Nash equilibrium when firms compete in price. Output competition results in an equilibrium price below the monopoly price and above marginal cost, while price competition results in the competitive solution.

More recently, authors have begun to consider the case where duopolists might choose different strategic variables, that is, a “Cournot–Bertrand” type model where one firm competes in output and the other competes in price. In the theoretical literature, Singh and Vives (1984) analyze Cournot, Bertrand and Cournot–Bertrand static duopoly models using the Dixit (1979) framework. When firms are given the choice of competing in output or price, Singh and Vives show that the dominant strategy is for both firms to compete in output when products are substitutes. However, recent work by Häckner (2000), Zanettin (2006), Arya et al. (2008), and Tremblay et al. (2009) shows that technological, institutional, and demand asymmetries can cause the Bertrand or Cournot–Bertrand outcomes to be optimal.

Kreps and Scheinkman (1983) argue that whether firms compete in output or price is ultimately an empirical question. In the real world, both Cournot and Bertrand behavior are observed. For example, farmers set quantities at local farmers’ markets, while restaurants set prices. Mixed Cournot–Bertrand behavior is also observed in the market for small cars, where Saturn and Scion dealers set prices and Honda and Subaru dealers set quantities (Tremblay et al., 2010). These empirical observations and recent theoretical developments indicate that the study of Cournot–Bertrand models would enhance our understanding of oligopoly markets.

Our goal is to investigate the influence of the degree of product differentiation on the static Cournot–Bertrand equilibrium. Specifically, we plan to explore stability, firm survivability, and the Nash equilibrium as the degree of product differentiation changes.1 The Cournot–Bertrand model with product differentiation is developed below.

2. The model

Two firms, 1 and 2, compete in the same market and make their strategic choices simultaneously. Firm 1 behaves as a Cournot-type firm by competing in output (q1). Firm 2 behaves as a Bertrand-type firm, competing in price (p2). Firms are profit maximizers, and information is complete.
Products differ on a number of characteristics, and consumers have a taste for variety as described by Beath and Katsoulacos (1991). A classic example is a small town with two restaurants, one serving Mexican food and the other serving Chinese food. These restaurants may differ in style of food, quality of service, atmosphere, and geographic location. Although consumers may prefer one style of food over the other, most consumers frequent both restaurants in a given period of time.

A model with this type of product differentiation produces a demand system that is similar to that of Bowley (1924), Dixit (1979) and Singh and Vives (1984)\(^2\):

\[
p_1 = a - q_1 - dq_2, \quad (1)
\]

\[
p_2 = a - q_2 - dq_1, \quad (2)
\]

where \(a > 0\) and \(d \in [0, 1]\). Products 1 and 2 are homogeneous when \(d = 1\), and each firm is a monopolist when \(d = 0\). Thus, \(d\) is an index of product differentiation, with differentiation increasing as \(d \rightarrow 0\). In the Cournot–Bertrand model, however, we write the demand system in strategic variables, \(q_1\) and \(q_2\):

\[
p_1 = \alpha - bq_1 + dp_2, \quad (3)
\]

\[
q_2 = a - p_2 - dq_1, \quad (4)
\]

where \(\alpha = a - ad\) and \(b = 1 - d^2\). Costs of production are linear and the same for both firms, and \(c\) equals marginal cost. Thus, the profit function for firm \(i\) is \(\pi_i = (p_i - c)q_i, \forall i = 1, 2\) and \(c \in (0, a)\).

In this model, firms face symmetric inverse demand and cost functions and differ only in their choice of strategic variable. The Nash equilibrium can be described in terms of the best-reply functions for each firm \((r_i)\):

\[
r_1 : p_2 = \frac{c + ad - a}{d} + 2\left(1 - d^2\right)q_1, \quad (5)
\]

\[
r_2 : p_2 = \frac{a + c - d}{2}q_1. \quad (6)
\]

Best-reply functions along with isoprofit functions are illustrated in Fig. 1. The Nash equilibrium occurs where the best-reply functions intersect, \(q_1^*\) and \(p_2^*\). Notice that the Pareto (shaded) region implies that both firms are better off at a higher \(p_2\) and a lower \(q_1\). The Nash equilibrium values are:

\[
p_1^* = \frac{a(2-d-2d^2 + d^3) + c(2 + d - d^2 - d^3)}{4 - 3d^2} > p_2^*
\]

\[
q_1^* = \frac{(a-c)(2-d)}{4 - 3d^2} > q_2^* = \frac{(a-c)(2 - d - d^2)}{4 - 3d^2}, \quad (8)
\]

\[
\pi_1^* = \frac{(a-c)^2(2-d)^2}{(4 - 3d^2)^2} > \pi_2^* = \frac{(a-c)^2(2-d-d^3)}{(4 - 3d^2)^2}. \quad (9)
\]

This outcome, which is similar to Singh and Vives (1984), has several interesting features. First, firm 1 has a strategic advantage over firm 2. Second, when \(d = 0\), the model confirms that firms behave as independent monopolists. Third, the equilibrium becomes more competitive (i.e., prices and profits fall) as product differentiation diminishes (i.e., as \(d\) increases).

The model has two other interesting features that are not discussed by Singh and Vives (1984). These are described in Propositions 1 and 2.

**Proposition 1.** With differentiated products, the equilibrium is stable when \(d < \left(\frac{\sqrt{3} - 1}{4}\right)\).

**Proof.** From Dixit (1986), a stable equilibrium in the Cournot–Bertrand model requires that \(|n_i| > |r_i|\), where \(n_i = \partial^2\pi_i/\partial q_i^2\), \(r_i = \partial^2\pi_i/\partial q_i\partial q_j\), \(s_1 = q_1\), and \(s_2 = q_2\). This implies that \(d(1 + 2d) - 2 < 0\) or that \(d < \left(\frac{\sqrt{3} - 1}{4}\right)\). Q.E.D.

Thus, when products are differentiated, the degree of differentiation must be sufficiently high to assure a stable equilibrium. According to Mas-Colell et al. (1995: 414), an equilibrium in a static model is stable when the “adjustment process in which the firms take turns myopically playing a best response to each others’ current
strategies converges to the Nash equilibrium from any strategy pair in a neighborhood of the equilibrium*. Fig. 1 provides an example where there is sufficient product differentiation to support stability. The equilibrium is unstable, however, when \( d = \left( \frac{\sqrt{a^2 - 4c}}{a} \right) \). This is illustrated in Fig. 2. In this case, if we start at a point such as \( p_2 \), firm 1’s best reply is to produce at the output level that corresponds to point A. In response to firm 1’s output decision, firm 2 sets price at B. Thus, the equilibrium is unstable because the adjustment process does not converge to the equilibrium (i.e., it moves from A to B, to C, to D, etc.).

It turns out, however, that the equilibrium is stable and the model produces an interesting outcome when products are perfectly homogeneous \( (d = 1) \).

**Proposition 2.** With perfectly homogeneous products, the equilibrium is stable, firm 2 produces zero output, firm 1 produces the perfectly competitive level of output, and the equilibrium price equals marginal cost.

**Proof.** This result derives directly from Eqs. (7) through (9) and the fact that the perfectly competitive level of output at the industry level is \( a - c \). Q.E.D.

The best-reply functions for this case are illustrated in Fig. 3. Notice that firm 1’s best-reply is horizontal at \( c \) and that firm 1 produces a level of output that meets total consumer demand when price equals marginal cost. This is much like that of a contestable market (Baumol et al., 1982). In the Cournot–Bertrand model, however, the threat of a Bertrand-type competitor that produces a homogeneous good ensures that a monopolist will behave as a perfectly competitive firm. This provides another example where a potential entrant dramatically reduces market power.

3 Notice that as \( d \) increases (differentiation diminishes), \( r_1 \) becomes flatter and \( r_2 \) becomes steeper.

### 3. Conclusion

The Cournot–Bertrand model has received little attention, even though real firms have been shown to compete in output and in price in a single industry. There is also theoretical evidence that technological and institutional forces can lead firms to choose different strategic paths. In light of this information, we attempt to provide a better understanding of the Cournot–Bertrand model. In particular, we analyze the influence of product differentiation on the equilibrium and its stability.

We find that the Cournot–Bertrand equilibrium with two firms is stable when there is sufficient product differentiation. As might be expected, a decrease in product differentiation leads to a more competitive equilibrium; that is, prices and profits fall. An interesting result emerges when products are perfectly homogeneous: only the Cournot-type firm survives, but it produces the perfectly competitive output at a price equal to marginal cost. In this setting, the presence of a potential Bertrand-type competitor guarantees a competitive outcome in an industry with only one firm.

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### References


