Balanced-budget income taxes and aggregate stability in a small open economy

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Abstract

It has been known that in a closed economy real business cycle model a balanced-budget fiscal policy rule with endogenous labor income tax rates can generate aggregate instability due to self-fulfilling expectations. Under such a balanced-budget rule, it is shown in this paper that in a small open economy under a perfect world capital market the steady state always exhibits saddle-path stability and hence beliefs-driven aggregate fluctuations can be ruled out. This result also holds in the presence of endogenous capital income taxes. From a policy perspective, the results of this paper suggest that if the fiscal authority relies on changes in income tax rates to achieve budget balance then an integrated world capital market can play a crucial role in stabilizing the economy.

JEL classification: E32; E62; F11; F4

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1 Introduction

A recurring topic of macroeconomic debate is whether a government should run a balanced-budget fiscal policy. Advocates of a balanced budget argue that persistent budget deficits can cause problems for both the economy and the government. In particular, the government’s borrowing associated with a budget deficit “crowds out” private investment spending and reduces the economy’s long run growth, and it further places financial pressure on the government’s future budgets and even its debt sustainability. Indeed, policy makers concerned about excessive deficits sometimes feel that rigid rules prohibiting - or at least setting an upper limit on - deficits are necessary. That was precisely what was stipulated in the Stability Pact in Europe in which it encouraged its member to follow the aim of balanced budgets.¹

There have been two main theoretical arguments against a balanced-budget rule. A traditional one is that it would amplify the business cycle by stimulating aggregate demand during booms via tax cuts or higher government expenditures and reducing it during recessions through fiscal contraction.² In a recent seminal contribution, Schmitt-Grohé and Uribe (1997, SGU henceforth) present an additional reason why a balanced-budget rule can be destabilizing. Specifically, they show that such a policy rule may induce indeterminacy and hence expectations-driven fluctuations, in a neoclassical model where the fiscal authority finances its expenditures with distortionary income taxes and labor income tax rates are endogenous.

The work of SGU (1997) has subsequently generated substantial research on balanced-budget rules and aggregate (in)stability.³ Furthermore, their results have led to a variety of policy recommendations on this issue. In particular, SGU (1997) suggest that if a balanced-budget rule is not to create endogenous aggregate instability then it should be coupled with restrictions on the fiscal authority’s ability to change tax rates. Guo and Harrison (2004) lend further support to SGU’s claim by showing that if the income tax rates are exogenous then self-fulfilling expectations cannot

¹As part of the agreement creating Euro, governments of member countries signed on to the Stability Pact. This agreement required each government to keep its budget deficit below 3% of the country’s GDP or face fines.
²As is well known, however, many economists and policy makers are also skeptical about the usefulness of counter-cyclical discretionary fiscal policy due to the time lags in its formulation and implementation.
occur. Giannitsarou (2007) argues that since consumption taxation ensures determinacy it should be preferred to income taxation, and that a stabilization target should increase the share of public spending financed by consumption taxation.\(^4\)

To our knowledge, existing studies on this issue in the literature including SGU (1997) all adopt closed economy frameworks. In this paper, we investigate the same issue studied in SGU (1997) in a neoclassical small open economy model, and examine how indeterminacy is relevant in such an environment. This is a realistic and necessary extension as most economies are not closed, but small and open.\(^5\) Indeed, under the same type of balanced-budget rule considered in SGU (1997), our main finding in this paper is that in a small open economy under a regime of perfect world capital markets indeterminacy cannot happen and that sunspot fluctuations can be ruled out. This result is in stark contrast to that obtained in SGU (1997). It suggests that an integrated global capital market can play a crucial role in stabilizing the economy under the balanced-budget fiscal policy, by preventing persistent endogenous aggregate fluctuations from happening.

Why can indeterminacy occur in SGU (1997) but otherwise cannot happen in a small open economy under the same balanced-budget rule? Consider the basic model in SGU (1997) where the public expenditures are assumed to be a constant and the fiscal authority levies a proportional tax on labor income. Taxation on labor income introduces a distortion, which affects the price of leisure in terms of the consumption good. Suppose that the households expect the tax rates to be higher. An increase in the tax rate would decrease the relative price of leisure, reducing both current consumption and labor supply through a substitution effect. But there is also an income effect induced by taxation. If the households expect their income to be lower, then they increase their current labor supply (i.e., to reduce leisure) and at the same time curtail their consumption.\(^6\) Thus the substitution effect and the income effect influence labor supply in opposite directions. Self-fulfilling expectations can happen in SGU (1997) because the substitution effect dominates.

\(^4\)However, recently, under a balanced-budget consumption tax, Nourry, et al. (2013) find that indeterminacy can still arise using a preference specification different from those used in Giannitsarou (2007), and Nishimura et al. (2013) show that indeterminacy can also occur in a two-sector model.

\(^5\)Even in the US, almost all the state governments follow the balanced budget rules.

\(^6\)As is noted in Wen (2001), in the literature it has been found difficult to provide a rigorous intuition about indeterminacy, and his paper provides an explanation based on households’ permanent income. The income effect which we refer to here has to do with the permanent income and is intertemporal. The intuition given in the present paper, though somewhat different, was also motivated by that provided in Giannitsarou (2007).
Since the labor tax rate is counter-cyclical, a reduction in labor supply would indeed render the tax rate to be higher, so that the households’ expectations are self-fulfilling.

The above economic intuition suggests that the final impact of a tax on labor income depends on the relative size of the substitution and income effects, and hence on the specification of the utility function. SGU (1997) adopts a separable utility function which is logarithmic in consumption, with a unit intertemporal elasticity of substitution in consumption. With such a preference, the increase in the households’ labor supply and reduction in consumption are small when expecting tax rate hikes. This is so because a large increase in labor supply and the corresponding increase in the investment goods would require a large drop in consumption, which is impossible when the intertemporal elasticity of substitution is small. Thus, in SGU (1997), the income effect is dominated by the substitution effect. However, as it is shown in this paper, if we replace in SGU (1997) by a general separable utility function with a constant intertemporal elasticity of substitution, then the relative size of the income and substitution effects can be reversed for sufficiently large intertemporal elasticity of substitution, which prevents households’ expectations from becoming self-fulfilling.

In a small open economy matters are very different. In expectation of higher tax rates, due to the income effect, the increase in the households’ labor supply and the associated investment good can be large, as the households can always borrow abroad to finance the needed investment without cutting their consumption, independently of the intertemporal elasticity of substitution in consumption. Thus, unlike in a closed economy, in an open economy where the households are allowed to borrow and lend internationally there is a de-linking between production decisions and consumption decisions. As the income effect dominates the substitution effect, labor supply increases, and the tax rates turn out to be lower, destroying self-fulfilling expectations.

The rest of the paper is organized as follows. In Section 2 we present a neoclassical closed economy model with two production sectors, a consumption good sector and an investment good sector, and in which the government follows the same type of balanced budget rule as in SGU (1997). The utility function is separable in consumption and leisure. The model collapses to the one sector model of SGU (1997) if the Cobb-Douglas production functions of the two sectors have the same form, and in particular their result can be replicated when the intertemporal elasticity of substitution in consumption is equal to 1 (that is, the utility is logarithmic in consumption).
However, in this one-sector model, we show analytically that given the values of all other parameters there exits an upper bound for the intertemporal elasticity of substitution, above which indeterminacy cannot occur, and the steady state generally exhibits saddle-path stability. Then it is shown that the same result holds if the production functions of the two sectors are different.

In Section 3 we then open the economy so that the households can borrow and lend internationally. For simplicity we assume that the consumption good is traded while the investment good is non-traded. In that setup we prove our main result: in the economy with a perfect world capital market indeterminacy can never happen and that the system generally exhibits saddle-path stability.

Section 4 demonstrates the robustness of the result of Section 3 in several extensions including cases with a general preference specification, with a capital income tax and with a traded investment good. Section 5 offers some concluding remarks.

2 The closed economy

The model

The production side of the closed economy consists of two sectors with one producing the consumption good \((y_{1t})\) and the other producing the investment good \((y_{2t})\). The production functions are assumed to be Cobb-Douglas with constant returns to scale:

\[
y_{it} = k_{it}^{\alpha_i} l_{it}^{1-\alpha_i}, \quad 0 < \alpha_i < 1, \quad i = 1, 2
\]  

(1)

where \(k_{it}\) and \(l_{it}\) are the capital and labor inputs utilized in the two sectors. Factor markets are perfectly competitive, and the first-order conditions for profit maximization imply that

\[
w_t = (1 - \alpha_1) \left( \frac{y_{1t}}{l_{1t}} \right) = p_t \left( 1 - \alpha_2 \right) \left( \frac{y_{2t}}{l_{2t}} \right)
\]  

(2)

\[
r_t = \alpha_1 \left( \frac{y_{1t}}{l_{1t}} \right) = p_t \alpha_2 \left( \frac{y_{2t}}{l_{2t}} \right)
\]  

(3)

where \(w_t, r_t\) are the pre-tax real wage rate and capital rental rate, and \(p_t\) is the relative price of investment to consumption goods.
The economy is inhabited by a large number of identical infinitely-lived households. The representative household maximizes its present discounted value of its lifetime utility

$$
\int_0^\infty \left( c_t^{1-\sigma} - \frac{l_t^{1+\chi}}{1+\chi} \right) e^{-\rho t} \, dt, \quad \sigma \geq 0, \quad \chi \geq 0, \quad \rho > 0
$$

(4)

where $c_t$ is consumption, $l_t$ is total hours worked, $\sigma$ is the relative risk aversion in consumption (it is equal to the reciprocal of the intertemporal elasticity of substitution), $\chi$ is the relative risk aversion in labor hours, and $\rho$ is the subjective discount rate. The household’s budget constraint is

$$
p_t i_t + c_t + T_t = w_t l_t + r_t k_t
$$

(5)

where $T_t$ is the labor income tax that the household pays to the government, and $i_t$ is the capital good investment.

The only source of government revenue is the labor income tax, and its expenditures are pre-set and constant. The government maintains a balanced budget such that

$$
G = T_t = T_{1t} + T_{2t} = \tau_t w_t l_{1t} + \tau_t w_t l_{2t} = \tau_t w_t l_t
$$

(6)

where $G$ denotes government purchases, $\tau_t$ is the labor tax rate, $T_{1t}$ is the tax on labor income in the consumption goods sector, and $T_{2t}$ is the tax on labor income on the investment goods sector.

The household’s budget constraint can be rewritten as

$$
p_t i_t + c_t = (1 - \tau_t) w_t l_t + r_t k_t
$$

(7)

The household’s total capital stock, denoted as $k_t$, evolves according to the law of motion

$$
\dot{k}_t = i_t - \delta k_t
$$

(8)

In addition, the market-clearing conditions for capital and labor imply that

$$
l_t = l_{1t} + l_{2t}, \quad k_t = k_{1t} + k_{2t}
$$

(9)

**Equilibrium and local (in)determinacy**

We solve the household’s optimization problem and give the equilibrium conditions in Appendix A.
The case of $\alpha_1 = \alpha_2$

When $\alpha_1 = \alpha_2 \equiv \alpha$, the model reduces to the one sector model (see Appendix A). However, the model here generalizes the one sector model in SGU (1997) in that we assume that the utility function has a constant intertemporal elasticity of substitution in consumption ($\sigma > 0$), whereas their utility function is logarithmic in consumption ($\sigma = 1$), which is a special case of our specification. It turns out that this generalization has important implications.

As in SGU (1997), it can be shown that the system can be reduced to the following two linearized dynamic equations:

$$
\begin{pmatrix}
\dot{\lambda}_t \\
\dot{k}_t
\end{pmatrix} =
J
\begin{pmatrix}
\lambda_t - \lambda^* \\
k_t - k^*
\end{pmatrix}
$$

(10)

where $\lambda_t$ is the costate variable associated with the budget constraint (7). The trace and determinant for the Jacobian matrix are

$$
\begin{align*}
Tr(J) &= \frac{\rho(\alpha + \chi)(1 - \tau^*) + \delta(1 - \alpha)\tau^*}{\kappa} \\
Det(J) &= -\frac{(\rho + \delta)^2 (1 - \alpha)^2 \Gamma(\tau^*)}{\alpha \sigma \kappa}
\end{align*}
$$

where $\kappa = \chi + \alpha - (1 + \chi)\tau^*$, and

$$
\Gamma(\tau^*) = (1 + \chi)(\tau^*)^2 - [\chi + (1 + \sigma + \chi)s]\tau^* + (\sigma + \chi)s = 0
$$

Here $s = \frac{\rho + \delta (1 - \alpha)}{(\rho + \delta)(1 - \alpha)} > 1$. As in SGU (1997), we can establish the following necessary and sufficient condition for indeterminacy of the perfect-foresight equilibrium.

**Proposition 1.** *In the closed economy where the government follows the balanced-budget labor income tax rule (6), if $\alpha_1 = \alpha_2 \equiv \alpha$, then the equilibrium is indeterminate if and only if*\textsuperscript{7}

$$
\tau < \tau^* < \tau^o,
$$

(11)

*where $\tau = \frac{\chi + \alpha}{1 + \chi}$, and $\tau^o \in (0, 1)$ is the solution to the quadratic equation*

$$
(1 + \chi)\tau^{2*} - [\chi + (1 + \sigma + \chi)s]\tau^* + (\sigma + \chi)s = 0
$$

(12)

\textsuperscript{7}In the reduced dynamic system (10), $\lambda_t$ is a jump variable and $k_t$ is a predetermined variable. Indeterminacy happens if the two roots of the Jacobian matrix are both stable. Saddle-path stability (determinacy) occurs if one root is stable and the other unstable. If the two roots are both unstable, then the system has no equilibrium solutions that converge to the steady state.
The case when $\sigma = 1$ and $\chi = 0$ corresponds to the benchmark model considered in SGU (1997). While indeterminacy occurs as long as (11) holds, it can be shown that for given values of all other parameters there exists a lower bound for $\sigma$ below which indeterminacy cannot happen. Specifically, we have the following corollary:

**Corollary 1.** For given values of all other parameters in the model, there exists a lower bound for $\sigma$, denoted as $\underline{\sigma}$, below which either the dynamic system exhibits saddle path stability or there are no equilibrium solutions, where

$$\underline{\sigma} \equiv \frac{(1 + \chi)(s - 1) + 1 - \alpha}{s} \frac{\alpha}{1 - \alpha} > 0$$

*Proof.* See Appendix B.

Note that as the intertemporal elasticity of substitution in consumption is the reciprocal of $\sigma$, the lower bound of $\sigma$ corresponds to the upper bound of the former, above which indeterminacy cannot occur.

For the parameter values used in the benchmark model in SGU (1997), $\rho = 0.04$, $\delta = 0.1$, $\alpha = 1/3$ and $\chi = 0$, we have $s = 1.143$, $\underline{\sigma} = 0.3542$. Obviously with logarithmic in consumption (i.e., $\sigma = 1$) in their paper, the lower bound condition for indeterminacy is satisfied.

The intuition for the results in Proposition 1 and Corollary 1 is as follows. Taxation on labor income introduces a distortion, which affects the price of leisure in terms of the consumption good. An increase in the tax rate would decrease the relative price of leisure, reducing both current consumption and labor supply through a substitution effect. But there is also an income effect induced by taxation. If the household expect their income to be lower due to higher tax rates, then they increase current labor supply (i.e., reduce leisure) to compensate for the income loss and curtail their consumption. So the substitution effect and the income effect influence current labor supply in opposite directions. In Proposition 1, and as in SGU (1997), self-fulfilling expectations can happen because the substitution effect dominates. Since the labor tax rate is counter-cyclical, a reduction in labor supply would indeed render the tax rate to be higher, so that the household’s expectations are self-fulfilling.

But this also suggests that the impact of a tax on labor income depends on the relative size of the substitution and income effects. If the intertemporal elasticity of substitution in consumption
is relatively small, as in SGU (1997), in expectation of tax rate hikes, the household is only willing to cut their consumption by a small amount: a large increase in labor supply is impossible as the corresponding increase in the capital goods would require a large drop in consumption. Thus, the income effect is dominated by the substitution effect. However, the relative size of the income and substitution effects can be reversed for sufficiently large intertemporal elasticity of substitution (i.e., sufficiently small \( \sigma \)). In such a case self-fulfilling expectations cannot happen.

**The case of \( \alpha_1 \neq \alpha_2 \)**

When \( \alpha_1 \neq \alpha_2 \), analytical results cannot generally be obtained. We solve the model numerically. The result on indeterminacy is similar to that of the case of \( \alpha_1 = \alpha_2 \). In Fig. 1, we illustrate the result by setting the same parameter values as in SGU (1997) except that we allow \( \alpha_2 \) to vary (it also includes the case of \( \alpha_1 = \alpha_2 \)).

![Fig. 1 about here](image1.jpg)

It is important to note that just as in the case of \( \alpha_1 = \alpha_2 \), for given values of all other parameters, there exists a lower bound for \( \sigma \) below which indeterminacy cannot occur and either the system exhibits saddle-path stability or there exist no equilibrium solutions that converge to the steady state. This can be seen from the numerical illustration in Fig. 2.

![Fig. 2 about here](image2.jpg)

Moreover, if \( \sigma = 0 \) (i.e., the intertemporal elasticity of substitution and the household is indifferent to consuming now or in the future), it can be analytically shown that determinacy always happens. In fact, in this case the linearized system becomes

\[
\begin{pmatrix}
\dot{\lambda}_t \\
\dot{k}_t
\end{pmatrix} = \begin{pmatrix}
\frac{(1-\alpha_2)(\rho+\delta)}{\alpha_1-\alpha_2} & 0 \\
-\alpha_1\alpha_2 \delta & -\frac{1-\alpha_1}{\alpha_1} \frac{k_1^*}{k_2} + \frac{1-\alpha_2}{\alpha_2}
\end{pmatrix} \begin{pmatrix}
\lambda_t - \lambda^* \\
k_t - k^*
\end{pmatrix}
\]

Here the two eigenvalues \( \frac{(1-\alpha_2)(\rho+\delta)}{\alpha_1-\alpha_2} \) and \(-\frac{1-\alpha_1}{\alpha_1} \frac{k_1^*}{k_2} + \frac{1-\alpha_2}{\alpha_2}\) have opposite signs, so the equilibrium is always determinate.

The basic intuition here of why indeterminacy cannot occur for sufficiently small \( \sigma \) remains the same as in the case of \( \alpha_1 = \alpha_2 \): the households are willing to change consumption and labor
supply by large magnitudes in expectation of tax hikes so that the income effect dominates. In the next section, we show that the dynamics of a small open economy facing perfect capital mobility resembles that of the closed economy model here with $\sigma = 0$.

3 The small open economy

We now assume that the economy is small and open, and that the domestic households are allowed to borrow and lend freely in a perfect world capital market. To facilitate the comparison between the results in this section to those in the previous section, we retain the same economic structure of a consumption good sector and an investment good sector. We assume that the consumption good is tradable and the investment good nontradable. In the next section we generalize the model to the case with both nontraded and nontraded investment goods.

As is well known in the literature on small open economy real business cycle models, the system would have a zero eigenvalue and hence nonstationary if, as in the closed economy model, the discount rate were assumed to be a constant.\(^8\) Various approaches have been introduced in the literature to resolve this problem; see, in particular, Schmitt-Grohé and Uribe (2003). Here we follow Bian and Meng (2004), which uses a variant of a method proposed in Schmitt-Grohé and Uribe (2003), by assuming that the discount rate is a function of the economy wide average level of consumption.\(^9\) Therefore, the representative household now maximizes its present discounted value of its lifetime utility

\[
\int_0^\infty \left( \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \frac{1}{1 + \chi} \right) e^{-\int_0^t \rho(\tau_s) d\tau_s} dt, \quad \sigma \geq 0, \quad \chi \geq 0
\]  

(13)

Here the instantaneous discount rate $\rho(\cdot)$ is a function that depends on the economy’s average level of consumption $\bar{c}_t$, which is taken as given by the household. Further, $\rho(\cdot)$ is an increasing function, i.e., $\rho'(\cdot) > 0$.

\(^8\)It is also known as the knife-edge property of the steady state. The zero root problem in continuous time models corresponds to the unit root problem in discrete time setups which are used in the literature on small open economy RBC models.

\(^9\)Specifying a discount rate that depends on the average levels of some variables instead of the individual variables can considerably simplify the analysis. See also Shi (1999) and Meng (2006) for other examples using similar approaches.
The households have access to net foreign bonds $b_t$, denominated in units of the tradable consumption good, that pays an exogenously given world interest rate $\theta$. The representative household’s budget constraint becomes

$$\dot{b}_t = \theta b_t + (1 - \tau_t) w_t l_t + r_t k_t - c_t - p_t i_t$$  \hspace{1cm} (14)$$

The household’s total capital stock evolves according to the law of motion in eq. (8). Eqs. (14) and (8) can be consolidated into

$$\dot{a}_t = \theta a_t + (1 - \tau_t) w_t l_t + r_t k_t - c_t + k_t (\dot{p}_t - \theta p_t - \delta p_t)$$  \hspace{1cm} (15)$$

where $a_t = b_t + p_t k_t$. Solving the household’s optimization problem yields the following first-order conditions

$$c_t^{\sigma} e^{-\int^t_0 \rho(s)ds} = \phi_t$$  \hspace{1cm} (16)$$

$$l_t^{\lambda} e^{-\int^t_0 \rho(s)ds} = (1 - \tau_t) \phi_t w_t$$  \hspace{1cm} (17)$$

$$\dot{\phi}_t = -\theta \phi_t$$  \hspace{1cm} (18)$$

$$\dot{p}_t = p_t \left( \theta + \delta - p_t^{-1} r_t \right)$$  \hspace{1cm} (19)$$

where $\phi_t$ is the costate variable associated with the budget constraint (15).

From eq. (2), we obtain

$$\frac{l_{2t}}{k_{2t}} = \eta p_t^{-\alpha_2 - \alpha_1} \equiv \psi (p_t)$$  \hspace{1cm} (20)$$

where $\eta = \left[ \left( \frac{\alpha_2}{\alpha_1} \right)^{\alpha_1} \frac{1 - \alpha_1}{1 - \alpha_2} \right]^{\frac{1}{\alpha_2 - \alpha_1}} > 0$.

Substituting the wage rate expression (2) into the government’s budget constraint (6) yields

$$G = (1 - \alpha_2) \tau_t p_t \psi_t^{-\alpha_2} l_t$$  \hspace{1cm} (21)$$

which, together with eq. (20), implies that

$$\eta^{\alpha_2} G \frac{\alpha_1}{1 - \alpha_2 p_t^{\alpha_2 - \alpha_1}} = \tau_t l_t$$  \hspace{1cm} (22)$$

Further, combining eqs. (20) and (17), we get

$$l_t^{N} = \left( 1 - \alpha_2 \right) \lambda_t (1 - \tau_t) p_t \left[ \psi (p_t) \right]^{-\alpha_2}$$  \hspace{1cm} (23)$$
From eqs. (23) and (22), \( l_t \) and \( \tau_t \) can be solved as functions of \( p_t \) and \( \lambda_t \).

Rearranging eq. (9), we can express \( k_{2t} \) as a function of \( \lambda_t, p_t \) and \( k_t \), i.e.

\[
k_{2t} = \frac{\alpha_1 (1 - \alpha_2) l(\lambda_t, p_t)}{\alpha_1 - \alpha_2} \psi(p_t) - \frac{(1 - \alpha_1) \alpha_2}{\alpha_1 - \alpha_2} k_t
\]

(24)

Note that \( l_t \) and \( \psi_t \) are both independent of \( k_t \).

Define \( \lambda_t = \phi_t e^\int_0^1 \rho(\tau_s)ds \). In equilibrium \( \bar{c}_t = c_t \). Thus, eq. (16) becomes

\[
c_t^{-\sigma} = \lambda_t
\]

and

\[
c_t = c(\lambda_t), \ c_\lambda < 0
\]

Finally, the market clearing condition for the investment good is given by

\[
y_{2t} = i_t + T_{2t}
\]

The system can be reduced to the following four dynamic equations

\[
\dot{p}_t = p_t \left[ \theta + \delta - \alpha_2 (\psi(p_t))^{1-\alpha_2} \right]
\]

(25)

\[
\dot{\lambda}_t = \lambda_t [\rho(c(\lambda_t)) - \theta]
\]

(26)

\[
\dot{k}_t = [1 - \tau_t (\lambda_t, p_t) (1 - \alpha_2)] [\psi(p_t)]^{1-\alpha_2} k_2(\lambda_t, p_t, k_t) - \delta k_t
\]

(27)

\[
\dot{b}_t = \theta b_t + [1 - \tau_t (\lambda_t, p_t) (1 - \alpha_1)] k_t^{\alpha_1} k_t^{1-\alpha_1} - c(\lambda_t)
\]

(28)

It is straightforward to show that the steady state exists and is unique for a given tax rate. The linearized system is given by

\[
\begin{pmatrix}
\dot{p}_t \\
\dot{\lambda}_t \\
\dot{k}_t \\
\dot{b}_t
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
* & * & 0 & 0 \\
* & * & * & 0
\end{pmatrix}
\begin{pmatrix}
p_t - p^* \\
\lambda_t - \lambda^* \\
k_t - k^* \\
b_t - b^*
\end{pmatrix}
\]

Thus the Jacobian matrix has a diagonal form, and its four eigenvalues are

\[
\begin{align*}
\dot{j}_{11} &= \frac{1 - \alpha_2}{\alpha_1 - \alpha_2} (\theta + \delta) \\
\dot{j}_{22} &= \lambda^* \rho' c_\lambda
\end{align*}
\]

12
\[ j_{33} = -\frac{\alpha_1 \alpha_2 \delta}{\alpha_1 - \alpha_2} \left( \frac{1 - \alpha_1 k_1^*}{\alpha_1 k_2^*} + \frac{1 - \alpha_2}{\alpha_2} \right) \]

\[ j_{44} = \theta \]

where

\[ \frac{k_1^*}{k_2^*} = \frac{[1 - \tau^* (1 - \alpha_2)] \theta + (1 - \tau^*) (1 - \alpha_2) \delta}{\alpha_2 \delta} \]

As \( j_{22} < 0, j_{44} > 0, j_{11} \) and \( j_{33} \) have opposite signs, and there are two jump variables and two predetermined variables, the system is always determinate.\(^{10}\)

**Proposition 2.** In the small open economy model, if the government follows the balanced-budget labor income tax rule (6), then the system exhibits saddle-path stability.

As we note in Section 2, in the closed economy, indeterminacy cannot occur when the intertemporal elasticity of substitution is large, as the income effect dominates the substitution effect. In a small open economy, self-fulfilling expectations cannot happen simply because the income effect dominates the substitution effect, independently of the intertemporal elasticity of substitution. In expectation of higher tax rates, the increase in the households’ labor supply and the needed investment on capital goods can be large, as the households can always borrow from the outside world to finance the new investment. As this effect dominates the substitution effect, labor supply increases, and the tax rates turn out to be lower, destroying self-fulfilling expectations.

The result here is in contrast to those of Weder (2001), Lahiri (2001), and Meng and Velasco (2003, 2004).\(^{11}\) In small open economy models with external effects in production, they all find that indeterminacy is more likely to arise under a perfect world capital market than otherwise in the closed economy. In fact, indeterminacy happens in their setups because of the strong external effects on the rental returns to physical capital. Expectations of higher future returns to capital call for an increase in current investment. In a closed economy with this kind of external effects, larger intertemporal elasticity of substitution in consumption actually makes it easier for indeterminacy

\(^{10}\)The analysis of this section focuses on the case of \( \alpha_1 \neq \alpha_2 \). In fact, for the case of \( \alpha_1 = \alpha_2 \), it can be shown that the dynamic system has a zero eigenvalue. However, one can introduce some kind of (even slight) asymmetry between the two sectors to resolve this problem. For example, if we assume that there exists a capital adjustment cost in one of the two sectors, or more generally, that the capital adjustment costs in the two sectors are different, then the system will have two positive eigenvalues and two negative ones with a steady state that is saddle path stable.

\(^{11}\)For other examples of open economy models with productive externalities and indeterminacy, see Nishimura and Shimomura (2002a, 2002b) and Hu and Mino (2013).
to occur, because if the households want to invest more they are more willing to adjust their consumption. In the presence of strong external effects in technology indeterminacy can happen more easily in these small open economy models because the households who want to investment more can borrow abroad without reducing their consumption level, independently of the intertemporal elasticity of substitution in consumption.

4 Extensions

This section demonstrates the robustness of the result of the small open economy model of Section 3 in several extensions. Section 4.1 examines an extension with a general preference specification. Section 4.2 adds capital income taxes to the model in Section 3, and Section 4.3 studies a small open economy model with both traded and nontradable investment goods.

4.1 The case with a general preference specification

Here we show that the main result of the benchmark model of Section 3 holds under a general preference specification. In particular, assume that the utility function is given by a general form

\[ u(c_t, 1 - l_t) \]  

(29)

Here \( u \) is concave, \( u_c > 0, u_l < 0 \), and consumption and leisure are normal goods. For the subjective discount rate, assume that it depends on both the average consumption \( (\bar{c}_t) \) and the average labor input \( (\bar{l}_t) \), i.e.,

\[ \rho = \rho (\bar{c}_t, \bar{l}_t), \rho_c \geq 0, \rho_l \leq 0, \rho_c^2 + \rho_l^2 \neq 0 \]  

(30)

The preference specification here is more general than that used in a model in Schmitt-Grohé and Uribe (2003, model 1a in their paper).

The first-order conditions for \( c_t \) and \( l_t \) are

\[ u_c = \lambda_t \]  

(31)

\[ -u_l = (1 - \alpha_2) \lambda_t (1 - \tau_t) p_t [\psi (p_t)]^{-\alpha_2} \]  

(32)

where \( \lambda_t = \phi_t e^{\int_0^t \rho (\bar{c}_s, \bar{l}_s) ds} \), and \( \phi_t \) is the costate variable associated with the household’s budget constraint. In equilibrium, \( \bar{c}_t = c_t \) and \( \bar{l}_t = l_t \), and we have the following the dynamic equation

\[ \dot{\lambda}_t = \lambda_t [\rho (c_t, l_t) - \theta] \]  

(33)
All other first-order conditions remain the same as the benchmark model of Section 3. It is also straightforward to show that the dynamic equations for $\dot{p}_t$, $\dot{k}_t$ and $\dot{b}_t$ remain the same as in the model of Section 3, while the dynamic equation for the costate variable now becomes

$$\dot{\lambda}_t = \lambda_t [p(c(\lambda_t, p_t), l(\lambda_t, p_t)) - \theta]$$

The reduced dynamic system also consists of four dynamic equations for $(p_t, \lambda_t, k_t, b_t)$. In this model, the Jacobian matrix of the linearized dynamic system takes the same diagonal form as in the model of Section 3, except that

$$j_{22} = \lambda^* (\rho_c c_\lambda + \rho_l l_\lambda)$$

In addition, the signs of the other three eigenvalues remain the same as in the model of Section 3, i.e., $j_{11}$ and $j_{33}$ have opposite signs, and $j_{44} > 0$. The system is determinate if $\lambda^* (\rho_c c_\lambda + \rho_l l_\lambda) < 0$. However, indeterminacy can never occur. The system has no equilibrium solutions if $\lambda^* (\rho_c c_\lambda + \rho_l l_\lambda) < 0$, which cannot be ruled out in this general model.

### 4.2 The case with both labor and capital income taxes

The main result of Section 3 holds if we add capital income tax. We consider here an extension of the model of Section 3 by assuming that there are both labor and capital income taxes, and their rates are denoted as $\tau^l_t$ and $\tau^k_t$ respectively, retaining the assumption that the discount rate depends only on the average level of consumption. The household’s budget constraint becomes

$$\dot{b}_t = \theta b_t + \left(1 - \tau^l_t\right) w_t l_t + \left(1 - \tau^k_t\right) r_t k_t - c_t - p_t i_t$$

(34)

The government expenditure follows the balanced-budget rule:

$$G = \tau^l_t w_t l_t + \tau^k_t r_t k_t$$

(35)

where $G$ is exogenous. When both labor and capital income taxes are endogenous and proportional, and labor is indivisible, SGU (1997) show numerically that indeterminacy still arises in their closed-economy model. In our small open economy model here, we can show analytical that indeterminacy can never arise. In this case, the model can be reduced to the following four dynamic equations:

$$\dot{p}_t = p_t \left[\theta + \delta - \alpha_2 \left(1 - \tau^k_t (\lambda_t, p_t)\right) (\psi(p_t))^{1-\alpha_2}\right]$$
\[ \dot{\lambda}_t = \lambda_t [\rho (c(\lambda_t)) - \theta] \]
\[ \dot{k}_t = \left[ 1 - \tau^l (\lambda_t, p_t) (1 - \alpha_2) \right] [\psi (p_t)]^{1-\alpha_2} k_2 (\lambda_t, p_t, k_t) - \delta k_t \]
\[ \dot{b}_t = \theta b_t + A_1 \left[ 1 - \tau^l (\lambda_t, p_t) (1 - \alpha_1) - \tau^k (\lambda_t, p_t) \alpha_1 \right] k_{1t}^{\alpha_1} l_{1t}^{1-\alpha_1} - c (\lambda_t) \]

The Jacobian matrix of the corresponding linearized system again has a diagonal form as in the model of Section 3. In particular, of the four eigenvalues, \( j_{22} < 0 \), and \( j_{44} > 0 \), and \( j_{11} \) and \( j_{33} \) are now given by

\[ j_{11} = -\frac{\theta + \delta}{(\alpha_2 - \alpha_1) (1 - \tau^{k*})} \left[ (1 - \alpha_2) \left( 1 - \tau^{k*} \right) + \alpha_1 \left( 1 - \tau^{l*} \right) \right] \]
\[ j_{33} = -\frac{k_1^* \tau^{l*} (1 - \alpha_1) + \tau^{k*} \alpha_1 + \tau^{k*} \alpha_2 + \tau^{l*} (1 - \alpha_2)}{\alpha_1 \alpha_2} \delta \]

As \( j_{11} \) and \( j_{33} \) always have opposite signs, the system exhibits saddle-path stability.

### 4.3 The case with both traded and nontraded capital goods

In this section, we generalize small open economy model of Section 3 in another dimension. In particular, we show that the results in Section 3 carry over to a setup with both traded and nontraded investment goods. Assume the traded goods sector now produces both the traded consumption good and a traded investment good such as equipment (denoted as \( e \)). The nontraded goods sector still produces the nontraded investment good such as structures (\( k \)). The production in each sector uses three factors: nontraded capital, traded capital, and labor. We demonstrate below that if the government follows the balanced-budget labor tax (6) then this extended model still exhibits saddle-path stability.

The production functions for the tradable goods \( (y_{1t}) \) and nontradable goods \( (y_{2t}) \) are Cobb-Douglas with constant returns to scale:

\[ y_{it} = k_{it}^{\alpha_i} e_{it}^{\beta_i} l_{it}^{1-\alpha_i-\beta_i}, \quad 0 < \alpha_i, \beta_i < 1, \quad i = 1, 2 \]

Under perfect competition for the factor markets, the real rental rates for the two capital goods and the real wage rates in the two sectors are

\[ r_{it}^k = \alpha_1 (y_{1t} / k_{1t}) = p_t \alpha_2 (y_{2t} / k_{2t}) \]
\[ r_t^e = \beta_1 (y_{1t}/e_{1t}) = p_t \beta_2 (y_{2t}/e_{2t}) \]
\[ w_t = (1 - \alpha_1 - \beta_1) (y_{1t}/l_{1t}) = p_t (1 - \alpha_2 - \beta_2) (y_{2t}/l_{2t}) \]

where
\[ k_t = k_{1t} + k_{2t}, \quad l_t = l_{1t} + l_{2t}, \quad e_t = e_{1t} + e_{2t} \]

The representative household maximizes its lifetime utility (13), by choosing consumption \( c_t \), labor supply \( l_t \), capital allocation decisions \( e_t, k_t \), rates of investment \( i_t^e, i_t^k \), and the rate of accumulation of bonds \( \dot{b}_t \), subject to
\[
\dot{b}_t = \theta b_t + (1 - \tau_t) w_t l_t + r_t^k k_t + r_t^e e_t - c_t - i_t^e - p_t i_t^k \\
\dot{e}_t = i_t^e - \delta^e e_t, \quad \dot{k}_t = i_t^k - \delta k_t
\]

We omit the technical details in solving the model. The system again reduces to four differential equations, and the Jacobian matrix of the linearized system also takes a diagonal form. In this case, \( j_{22} < 0 \), and \( j_{44} > 0 \), and \( j_{11} \) and \( j_{33} \) are given by
\[
j_{11} = \frac{\alpha_1 (1 - \alpha_2 - \beta_2) + (1 - \alpha_2) (1 - \alpha_1 - \beta_1)}{\alpha_1 (1 - \beta_2) - \alpha_2 (1 - \beta_1)} (\theta + \delta) \\
j_{33} = -\frac{\alpha_1 \alpha_2 \delta}{\alpha_1 (1 - \beta_2) - \alpha_2 (1 - \beta_1)} \left( \frac{1 - \alpha_1 - \beta_1}{\alpha_1} \frac{k_t^*}{k_1^*} + \frac{1 - \alpha_2 - \beta_2}{\alpha_2} \right)
\]

which always have opposite signs. Therefore, in this small open economy with both traded and nontraded capital, if the government follows the balanced-budget labor income tax rule (6), then the system exhibits saddle-path stability.

In summary, the main result of this paper is robust to a variety of extensions. For all the cases, the intuition remains the same as in the benchmark model in Section 3: because of the existence of a perfect world capital market, the households are able to increase their labor supply and the capital goods investment by large magnitudes in expectation of tax hikes, which prevents households’ expectations from becoming self-fulfilling.

5 Conclusion

This paper revisits the issue of aggregate instability arising from self-fulfilling expectations under a balanced-budget income tax rule, in a neoclassical small open economy model. The analysis
shows that for a small open economy under perfect capital mobility saddle-path stability always prevails and that self-fulfilling expectations can be ruled out. This is in contrast to the results obtained in a closed economy model. The reason is that, unlike in a closed economy, in an open economy where the households are allowed to borrow and lend internationally there is a de-linking between production decisions and consumption decisions. From a policy perspective, the results of this paper suggest that if the fiscal authority relies heavily on changes in income tax rates to achieve budget balance an integrated global capital market can play a crucial role in stabilizing the economy.

The research topic of this paper can be explored further in several directions. First, it can be extended to study the (in)stability effects of the balanced-budget rules based on other types of policies such as consumption taxation. Second, the determinacy results of this paper are all based on local analyses. As Stockman (2010) has shown in a closed economy model à la SGU (1997), global indeterminacy can still occur even if there is local determinacy. So it would be worthwhile to conduct a global analysis to examine the robustness of the local results. Finally, it would be interesting to study a two-country world model, and to investigate the effects of the balanced-budget fiscal policies by taking into account the interdependence between nations. We plan to pursue these extensions in the near future.
Appendix A: Equilibrium solution for the closed economy model

We briefly solve the two-sector closed economy model here. The Hamiltonian is given by

$$H = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \frac{1}{1 + \chi} l_t^{1+\chi} + \lambda_t (i_t - \delta k_t) + \phi_t [(1 - \tau_t) w_t l_t + r_t k_t - p_t i_t - a_t]$$

The first-order conditions are

$$c_t^{-\sigma} = p_t^{-1} \lambda_t$$

$$l_t^\lambda = (1 - \tau_t) w_t p_t^{-1} \lambda_t$$

$$\dot{\lambda}_t = \lambda_t (\rho + \delta - p_t^{-1} r_t)$$

Also from the market clearing conditions, we have

$$c_t = k_t^{\alpha_1} l_t^{1-\alpha_1} - T_1 = [1 - \tau_t (1 - \alpha_1)] \left( \frac{l_t}{k_t} \right)^{1-\alpha_1} k_t$$

$$i_t = k_t^{\alpha_2} l_t^{1-\alpha_2} - T_2 = [1 - \tau_t (1 - \alpha_2)] \left( \frac{l_t}{k_t} \right)^{1-\alpha_2} k_t$$

By making use of eqs. (2), (3), (6) and (20), we can reduce the system to the following two equations

$$\dot{\lambda}_t = \left\{ \rho + \delta - \alpha_2 \eta^{1-\alpha_2} [p (\lambda_t, k_t)]^{1-\alpha_2} \alpha_2^{-\alpha_1} k_2 (\lambda_t, k_t) \right\} \lambda_t$$

$$\dot{k}_t = \eta^{1-\alpha_2} [1 - \tau (\lambda_t, k_t) (1 - \alpha_2)] [p (\lambda_t, k_t)]^{1-\alpha_2} \alpha_2^{-\alpha_1} k_2 (\lambda_t, k_t) - \delta k_t$$

When $\alpha_1 = \alpha_2 = \alpha$, the model reduces to the one sector model. To see this, notice that if $\alpha_1 = \alpha_2$, we have

$$p_t = 1$$

$$\frac{l_t}{k_t} = \frac{l_t}{k_t} \equiv \psi_t$$

and

$$l_t = \psi_t k_t$$

The first-order conditions now become

$$c_t^{-\sigma} = \lambda_t$$

$$l_t^\lambda = (1 - \tau_t) (1 - \beta) \lambda_t \kappa_t \alpha_t^{1-\alpha}$$

$$G = (1 - \alpha) \tau_t \kappa_t \alpha_t^{1-\alpha}$$
These first-order conditions are the same as those in the one-sector closed economy model. Further, the market clearing conditions can be written as

\[ c_t = \left[1 - \tau_t (1 - \alpha)\right] \left(\frac{l_t}{k_t}\right)^{1-\alpha} (k_t - k_{2t}) \]

\[ i_t = \left[1 - \tau_t (1 - \alpha)\right] \left(\frac{l_t}{k_t}\right)^{1-\alpha} k_{2t} = y_t - c_t - G \]

where

\[ y_t = k_t^{\alpha} l_t^{1-\alpha} \]

Therefore, the two dynamic equations can be reduced to

\[ \dot{\lambda}_t = (\rho + \delta - \alpha k_t^{\alpha-1} l_t^{1-\alpha}) \lambda_t \]

\[ \dot{k}_t = y_t - \delta k_t - c_t - G \]

which are identical to those in the one-sector closed economy model.

**Appendix B: Proof of Corollary 1**

We prove Corollary 1 here. Note that \( \tau^o \) must lie inside the interval of \((\tau, 1)\) in order for the indeterminacy condition to hold. As \( \Gamma(1) = \frac{1-s}{\sigma + \chi} < 0 \) and \( \tau < 1 \), indeterminacy requires that \( \Gamma(\tau) > 0 \), which immediately implies that there exists a lower bound for \( \sigma \) (denoted as \( \sigma^* \)) that is necessary for indeterminacy, i.e.

\[ \sigma > \sigma^* \equiv \frac{(1 + \chi)(s - 1) + 1 - \alpha}{s} \frac{\alpha}{1 - \alpha} > 0 \] \hspace{1cm} (.36)

If \( \sigma < \sigma^* \), then \( \Gamma(\tau) < 0 \) and \( \Gamma(0) > 0 \), and there exists a \( \tau^o \in (0, \tau) \). The following cases are possible:

(i) If the steady state value \( \tau^* \in (\tau, 1) \) (which is the necessary condition for \( Tr(J) < 0 \)), then \( \Gamma(\tau^*) < 0 \) and \( D(J) < 0 \). The system is always determinate.

(ii) If \( \tau^* \in (0, \tau) \) (i.e., \( \frac{\tau^* - \alpha}{1 - \alpha} < \chi \)), then \( Tr(J) > 0 \). Either the system is determinate if \( 0 < \tau^* < \tau^o \) (as \( D(J) < 0 \)) or there exist on equilibrium solutions \( \tau^o < \tau^* < \tau \) (as \( D(J) > 0 \)).

Thus, for \( \sigma < \sigma^* \), the system is determinate if \( \tau^* \in (0, \tau^o) \cup (\tau, 1) \), and there are no equilibrium solutions if \( \tau^* \in (\tau^o, \tau) \).
References


Fig. 1. The two-sector closed economy model ($\alpha_1 = 1/3$): D – determinacy (saddle-path stability), I – indeterminacy, N – no equilibrium
Fig. 2. The two-sector closed economy model \((\alpha_1 = 1/3)\): lower bounds of \(\sigma\) for indeterminacy for cases of \(\alpha_1 > \alpha_2\) or \(\alpha_1 < \alpha_2\), and when \(\chi = 0\) or 1.