Market imperfections and the instability of open economies

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Abstract

In the presence of small market imperfections, the transitional dynamics of an open economy can become indeterminate, in that there exist an infinite number of equilibrium paths converging to a unique steady state. In contrast to closed economy models, in the open economy, such indeterminacy can arise independently of the curvature of the utility function in consumption. The results suggest that with market imperfections, open economies can be subject to fluctuations caused by randomness unrelated to the economy’s fundamentals.

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1. Introduction

Are macroeconomic equilibria unique? A growing literature argues they are not and explores the possibility of indeterminacy and sunspots in dynamic general equilibrium economies with market imperfections.\textsuperscript{1} Indeterminacy means that from the same initial condition there exist an infinite number of equilibria, all of which converge to a common steady state. This allows for the existence of sunspot equilibria—that is, equilibrium

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\textsuperscript{1} See Benhabib and Farmer (1999) for a survey paper and also the treatise by Farmer (1999).

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allocations influenced by purely extrinsic beliefs unrelated to the economy’s fundamentals (see, e.g., Cass and Shell, 1983; Woodford, 1986). In turn, such sunspot equilibria provide a modern interpretation of Keynes’s hypothesis that economic fluctuations are driven by the “animal spirits” of businessmen.

Most models in the indeterminacy literature are closed economy. This has made it hard to satisfy the conditions necessary for generating indeterminacy. One issue has to do with technology. While early models relied on large increasing returns or large external effects to generate indeterminacy (e.g., Benhabib and Farmer, 1994), recent theoretical work, in particular in multisector models, demonstrates that only small market imperfections are required. The other issue has to do with preferences. These closed-economy models also require restrictions on the curvature on the utility function to generate indeterminacy. With small market imperfections, the models can only generate indeterminacy when the intertemporal elasticity of substitution in consumption is high—indeed, in some cases, the utility function has to be linear or close-to-linear in consumption. In short, there exists a tradeoff between the size of market imperfections and the magnitude of intertemporal elasticity of substitution needed for indeterminacy.

The intuition for this tradeoff is easy to understand. Suppose there are two sectors in the closed economy: a consumption good sector and an investment good sector. Indeterminacy occurs if, while going along an equilibrium path, the representative agent decides to invest more and to jump onto an alternative path—and this turns out also to be an equilibrium, in that asset prices and returns then move in such a way as to make the jump optimal. But in the closed economy, for the agent to invest more, he must first curtail consumption. If the elasticity of intertemporal substitution in consumption is sufficiently low, doing so will be very costly, and the desire to smooth consumption may dominate the incentive to invest more, making the existence of an alternative equilibrium path impossible.

In this paper, we focus on a small open economy model and investigate how and when indeterminacy can occur. Our main finding is that in an open economy, the conditions for indeterminacy can be satisfied more easily than in a closed economy. We show that in a two-sector small open economy with perfect access to a world bond market, indeterminacy can occur under very small or even negligible market imperfections, for technologies that exhibit constant marginal costs, and independently of the curvature of utility in consumption. These results suggest that in the presence of market imperfections, small open economies can be vulnerable to fluctuations caused by extrinsic uncertainty unrelated to the economy’s fundamentals.

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2 For recent theoretical papers in this area, see, for example, Benhabib and Farmer (1996) and Benhabib and Nishimura (1998). While empirical evidence on the size of returns to scale remains controversial, a number of researchers have recently found that returns to scale seem to be roughly constant and that market imperfections are small. See Basu and Fernald (1997) and Burnside et al. (1995), whose findings are in contrast to those by Hall (1988b) and others.

3 Linear or close-to-linear utility in consumption runs counter to most empirical estimates on elasticities of intertemporal substitution. See, e.g., Hall (1988a).

4 As will be clear in the next section, the externalities or market imperfections required for indeterminacy can be arbitrarily small.
The reason that indeterminacy can arise more easily in open economy models is straightforward. The proverbial representative agent who considers investing more and jumping to another equilibrium trajectory can do so without having to reduce consumption since he can always finance additional investment by borrowing from the outside world. The curvature of the utility function does not affect investment decisions, and indeterminacy can occur for any degree of intertemporal substitution. Only minimal market imperfection conditions in technology have to be satisfied.

In all the examples of indeterminacy we construct below, the dynamic paths for investment and consumption are not unique, and therefore the current account and capital flows that finance it are also indeterminate. If sunspots move the economy from one equilibrium trajectory to another, then the capital and current accounts—and the associated prices—can be subject to sudden and potentially large movements governed by self-fulfilling prophecies. Such results are reminiscent of concerns often voiced in the policy literature: confronted with volatile capital flows and the resulting fluctuations in relative prices, analysts fret over the consequences and worry that such fluctuations may be the result of self-fulfilling prophecies. Some go as far as to advocate restrictions or at least gradualism in deregulating the capital account. Yet the theoretical foundations for such concerns are not always clear.

To our knowledge, the first paper that studies indeterminacy in the open economy is Lahiri (2001). Unlike us, Lahiri is concerned with multiple growth paths in an endogenous growth model. In addition, his model relies on increasing returns (or decreasing marginal costs). Weder (2001) is closer in spirit to the present paper. But his model also depends on the unrealistic assumption of decreasing marginal costs to generate indeterminacy, while ours does not. Unlike Weder, we can also allow for traded capital in our framework. In addition, besides externalities we also analyze the effects of other kinds of market distortions—like factor taxation—in generating equilibrium indeterminacy.

The paper is organized as follows. In Section 2, we consider a two-sector small open economy with production externalities. For simplicity, we assume that the consumption good is traded and the capital good is nontraded. In this setup we prove our main result: for the small open economy facing a perfect world bond market, indeterminacy can occur regardless of the degree of intertemporal substitution in consumption. Only a technological condition on the sectoral factor intensities is required. In Section 3, we extend the model in Section 2 in two directions. Our first extension shows that including traded capital does not alter the main result, and that a similar condition on the sectoral factor intensities is all that is needed for indeterminacy. In the second extension, we demonstrate that externalities are not necessary to obtain the indeterminacy result, and that other market distortions like factor taxation have the same effect. Section 4 concludes.

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5 For some recent policy thinking on capital account liberalization and its consequences, see Eichengreen et al. (1998) and Fischer et al. (1998).

6 The model can be regarded as an open economy version of the two-sector closed-economy model presented in Benhabib and Nishimura (1998).
2. The two-sector open economy with nontraded capital

2.1. The model

Consider a small open economy inhabited by an infinite-lived representative agent, who is endowed with perfect foresight and maximizes the intertemporal utility function

\[ U = \int_0^\infty u(c_t)e^{-\rho t}dt, \]  

where \( c_t \) is the consumption of traded goods and \( \rho \) is the parameter of time preference. In this section, we assume that the consumption good is tradable and the capital good is nontraded. The domestic representative agent faces a perfect world bond market and can therefore borrow from and lend to the outside world freely. Net foreign bonds \( d_t \), denominated in units of the tradable good, pay an exogenously given world interest rate \( r \).

On the production side, there are two sectors, with one producing the tradeable consumption good \((y_{1t})\) and the other the nontradeable capital good \((y_{2t})\). The production functions for the two sectors are Cobb–Douglas with capital-generated externalities and constant social returns:

\[ y_{1t} = l_{1t}^a k_{1t}^a \bar{k}_1, \quad y_{2t} = l_{2t}^b k_{2t}^b \bar{k}_2, \]

where

\[ k_{1t} + k_{2t} = k_t, \quad l_{1t} + l_{2t} = 1, \]

and \((a + x_1 + a) = (b + b_1 + b) = 1\).\(^7\) Note that labor supply is inelastic and normalized to unity. The externality components in the production functions involve the average levels of the capital stock in each sector, and the parameters \( a \) and \( b \) denote the degree of capital externalities. The assumption of capital externalities here follows the tradition in the growth literature and can be justified through learning-by-doing through firms’ investment (or learning-by-investing).\(^8\) The private agent who solves this optimization problem takes the average levels of capital stock in each sector as given, though in equilibrium they are of course equal to equilibrium values.

The market structure implied by the production functions in Eq. (2) merits further explanation. Constant social returns coupled with small external effects mean that some sectors must display a small degree of decreasing returns at the private level. Such a market structure exhibits increasing private marginal costs but constant social marginal costs, which is in line with current empirical evidence. This is in contrast to models with social increasing returns and private constant returns, popular in the literature.

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\(^7\) The assumption of social constant returns to scale follows Benhabib and Nishimura (1998) and Benhabib et al. (2000).

\(^8\) See, e.g., Arrow (1962) and Romer (1986).
An implication of decreasing private returns is positive profits. Unless the number of firms is fixed, one must assume that there is a fixed entry cost to determine the number of firms along the equilibrium trajectories. As is clear from the example and propositions below, the external effects and the degree of decreasing returns required for the indeterminacy results may be arbitrarily small and therefore generate only tiny profits. If the discounted value of profits along the equilibrium trajectories converging to the steady state is small, a small fixed entry cost will be sufficient to deter new entrants.\footnote{For example, a producer in the traded goods sector pays a fixed entry cost, }\[ F = \int_0^\infty e^{-\rho t} \text{(profit at time } t) \, dt = (1 - x_0) \int_0^\infty e^{-\rho t} (l_{1t} k_{1t})^{-z} \, dt, \] where the integral is evaluated along the equilibrium trajectories. After incurring this one-time entry cost, he then produces using the technology \[ y_{1t} = k_{1t} l_{1t}^z, \] which is equal to \[ l_{1t} k_{1t}^{-z} \] along the equilibrium trajectories. Since he makes a profit of \((1 - x_0)(l_{1t} k_{1t})^{-z}\) at any time \(t\), the present value of the total profits net of the entry cost \(F\) is zero. With the fixed entry cost, therefore, the production function exhibits increasing returns in the dynamic sense, which is sometimes referred to as “dynamic increasing returns” in similar situations. See, e.g., Ciccone and Matsuyama (1996) (through a startup cost) and Young (1993) (through a patent cost).

The agent’s budget constraint is
\[ d_t = r d_t + y_{1t} + p_t y_{2t} - c_t - p_t i_t, \] where \(p_t\) is the price of the investment good in terms of the consumption good. The agent is also constrained by the standard no-Ponzi game condition
\[ \lim_{t \to \infty} d_t e^{-\rho t} \geq 0. \] The variable \(i_t\) denotes gross investment, so that the law of motion for capital is
\[ \dot{k}_t = i_t - \delta k_t, \] where \(\delta\) is the depreciation rate. Eqs. (4) and (6) can be consolidated into
\[ z_t = r z_t + y_{1t} + p_t y_{2t} - c_t + k_t (\dot{p}_t - r p_t - \delta p_t), \] where \(z_t = d_t + p_t k_t\). The agent’s problem is to choose \(c_t, l_{1t}, l_{2t}, i_t, k_{1t}, k_{2t}\) and \(d_t\) to maximize utility, given by Eq. (1), subject to the production functions in Eq. (2) and the constraints in Eqs. (3) and (7), given \(k_0\) and \(d_0\).

2.2. Solution and equilibrium conditions

The Hamiltonian is
\[
\mathcal{H} = u(c_t) + \lambda_t [r z_t + y_{1t} + p_t y_{2t} - c_t + k_t (\dot{p}_t - r p_t - \delta p_t)] + \mu_t (k_t - k_{1t} - k_{2t}) + \omega_t (1 - l_{1t} - l_{2t}),
\]
where $\lambda_t$ is the costate variable and $\mu_t$ and $w_t$ are the rental rate of the capital good and the wage rate of labor, respectively, all in terms of the consumption good. First-order conditions for maximization are:

1. $u'(c_t) = \dot{\lambda}_t,$ (8)
2. $\mu_t = \lambda_t \alpha_1 l_{1t}^{\alpha_1} k_{1t}^{1-\alpha_1} = \dot{\lambda}_t \beta_1 p_t l_{2t}^{\beta_1} k_{2t}^{1-\beta_1},$ (9)
3. $w_t = \lambda_t \alpha_2 l_{1t}^{\alpha_2-1} k_{1t}^{1-\alpha_2} = \dot{\lambda}_t \beta_0 p_t l_{2t}^{\beta_0-1} k_{2t}^{1-\beta_0},$ (10)
4. $\dot{\lambda}_t = \lambda_t (\rho - r),$ (11)
5. $\dot{p}_t = p_t (r + \delta - \beta_1 l_{1t}^{\beta_1} k_{2t}^{\beta_1}),$ (12)
6. $\lim_{t \to \infty} \lambda_t d_t e^{-\rho t} = \lim_{t \to \infty} \lambda_t p_t k_t e^{-\rho t} = 0.$ (13)

As is standard in international macroeconomics, we impose $\rho = r$, a condition that ensures a well-defined steady state with constant bond holdings. This assumption also implies, by Eq. (11), that marginal utility remains constant over all time—that is, $\lambda = \bar{\lambda}$. Substituting $\lambda = \bar{\lambda}$ into other first-order conditions, by Eq. (8), we have

$$c_t = c_t(\bar{\lambda}) = \bar{c},$$ (14)

which means that consumption is completely smoothed.10 Dividing Eq. (9) by Eq. (10) gives

$$\frac{\alpha_1 l_{1t}}{\alpha} k_{1t} = \frac{\beta_1}{\beta} l_{2t}.$$ (15)

Using Eqs. (9) and (15) to solve for $l_{2t}/k_{2t}$, we obtain

$$\frac{l_{2t}}{k_{2t}} = \xi p_t^{-\beta_1} g(p_t),$$ (16)

where $\xi = (\beta_1 / \alpha_1)(\alpha_1 / \alpha_1)^\alpha > 0$. Solving for $k_{2t}$ from Eqs. (3), (15) and (16) yields

$$k_{2t} = \left( \frac{\alpha_1}{\alpha_1 / \beta_1 - \alpha_1} \right) k_t - \left( \frac{\alpha_1 / \beta_1}{\alpha_1 / \beta_1 - \alpha_1} \right) \frac{1}{g(p_t)}.$$ (17)

10 Not imposing $\rho = r$ would mean that bond-holdings and consumption would not be constant in the steady state. Under some conditions (see, e.g., Rebelo, 1992), this can be an allowable equilibrium trajectory. But the additional complications arguably would add little to the point that we are trying to make. In particular, it is true that the speed of asset accumulation or decumulation along the steady state would depend on the elasticity of intertemporal substitution, but the possibility of indeterminate dynamics leading to that steady state would not.
In addition, the market-clearing condition for the capital good is given by
\[ \dot{k}_t = \ln \frac{h_t}{k_t} - \delta k_t. \] (18)

Substituting Eqs. (16) and (17) into Eqs. (12) and (18), we obtain the following dynamic equations for \( k_t \) and \( p_t \):
\[ \dot{p}_t = p_t [r - \delta - \beta_1 g(p_t)\beta], \] (19)
\[ \dot{k}_t = \left[ \frac{\alpha \beta_1 g(p_t)\beta}{\alpha \beta_1 - \alpha \beta} - \delta \right] k_t - \frac{\alpha \beta g(p_t)\beta - 1}{\alpha \beta_1 - \alpha \beta} k_t. \] (20)

Next, we solve for the other variables in the model. From the budget constraint Eq. (4), the economy’s current account is given by
\[ \dot{d}_t = rd_t + y_{1t}(p_t, k_t) - \bar{c}, \] (21)
where we have written \( y_{1t} = \int \frac{(1 + \delta - \alpha \beta g(p_t)\beta)}{(\alpha \beta_1 - \alpha \beta)} \) as a function of \( p_t \) and \( k_t \), which follows from Eqs. (15)–(17). Integrating over Eq. (21), and using the transversality condition (13), we have
\[ \bar{c} = \int_0^\infty y_{1t}(p_t, k_t)e^{-rt}dt - d_0. \] (22)

Note that the dynamic system characterizing the behavior of the economy is block-recursive. In particular, the differential equations governing the dynamics for \( p_t \) and \( k_t \), Eqs. (19) and (20), are independent of consumption and foreign bond holdings. In other words, the investment decision is independent of the consumption decision. The system can be solved by first determining \( p_t \) and \( k_t \) from Eqs. (19) and (20); consumption and foreign bound dynamics are then determined by Eqs. (21) and (22). In particular, \( d_t \) is given by integrating Eq. (21) from 0 to \( t \), i.e.,
\[ d_t = e^{rt} \left[ \int_0^t y_{1t}(p_t, k_t)e^{-rt}dt - \frac{\bar{c}}{r}(1 - e^{-rt}) + d_0 \right]. \] (23)

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11 One can show that \( y_{1t}(p_t, k_t) = (\alpha \beta_1 / \alpha \beta)(1 + \delta - \alpha \beta g(p_t)\beta)k_t + \alpha \beta((\alpha \beta_1 - \alpha \beta)). \)

12 Notice that since the marginal utility of consumption is constant along the equilibrium path, condition (13) implies that Eq. (5) holds with equality.

13 The block-recursive feature of our model is similar to the one-sector Ramsey-type small open economy model in Blanchard and Fischer (1989, Chapter 2). There, a capital adjustment cost is introduced to ensure well-defined dynamics for investment. In this paper, we do not need such adjustment cost; it is the presence of an additional sector (nontraded capital sector) that guarantees nondegenerate dynamics.
2.3. Steady state

From Eqs. (19) and (20), it is clear that there exists a unique steady state for \( p_t \) and \( k_t \), \((p^*, k^*)\), given by:

\[
p^* = \left[ \frac{r + \delta}{\beta \xi^\beta} \right]^{\frac{\beta - \delta}{\beta}} > 0,
\]

\[
k^* = \frac{\beta - 1}{\alpha r + \alpha \beta \delta + \alpha \delta (1 - \beta_1)} > 0.
\]

The steady state for \( d_t \) is given by setting the right-hand side of Eq. (21) to 0. That is,

\[
d^* = \frac{1}{r} \left[ \tilde{c} - y_1(p^*, k^*) \right] = \left[ \int_0^\infty y_1(p_t, k_t)e^{-rt} dt - d_0 \right] - \frac{1}{r} y_1(p^*, k^*) \]

In addition, the constant consumption level \( \tilde{c} \), given in Eq. (22), is also the steady-state level for consumption.

Note that while the steady state for consumption or bonds depends on initial conditions, the steady state for the relative price or nontraded capital does not. This feature of the model is in sharp contrast to other one-sector real business cycle models with both an exogenous discount rate and an exogenous interest rate, where the dynamic system is generically not block-recursive and the steady state for each variable depends on initial conditions.

2.4. Dynamics and indeterminacy

We next turn to the model’s dynamics. Because the dynamic system is block-recursive, we can focus on the subsystem for \( p_t \) and \( k_t \). From Eqs. (22) and (23), equilibrium uniqueness (or indeterminacy) for \( p_t \) and \( k_t \) implies uniqueness (or indeterminacy) for consumption level and foreign bond holdings and hence the overall economy. In particular, if there is indeterminacy for \((p_t, k_t)\), indeterminacy for consumption and bond holdings then follows because the integrals in Eqs. (22) and (23) are evaluated along each of the indeterminate transitional paths for \((p_t, k_t)\).

Linearizing Eqs. (19) and (20) around the steady state, \( p^* \) and \( k^* \) given in Eqs. (24) and (25), the dynamics of \( k_t \) and \( p_t \) can be approximated by the system

\[
\begin{pmatrix}
\dot{p}_t \\
k_t
\end{pmatrix} = \begin{pmatrix}
J_{11} & 0 \\
J_{21} & J_{22}
\end{pmatrix} \begin{pmatrix}
p_t - p^* \\
k_t - k^*
\end{pmatrix},
\]

where

\[
J_{11} = \frac{\beta (r + \delta)}{(\beta - \alpha)} = \frac{\beta (r + \delta)}{(\alpha \beta - \alpha \beta_1) + (\alpha \beta - \alpha \beta_1)}.
\]
and

\[ j_{22} = \frac{\alpha r + \alpha_1 \beta \delta + \alpha \delta (1 - \beta_1)}{(\alpha \beta_1 - \alpha_1 \beta)} \]  

(29)

The relevant eigenvalues of the Jacobian matrix are \( j_{11} \) and \( j_{22} \), whose signs are related to the sectoral factor intensity differences from the social and private perspective, respectively. Notice also that \( k_t \) is a predetermined variable and evolves continuously, while \( p_t \) is a nonpredetermined variable—that is, its initial value is not given by history and instead responds to new information. The existence of a unique saddle path requires one eigenvalue to be positive and the other negative. Two negative eigenvalues mean equilibrium indeterminacy, while two positive ones imply that no equilibrium trajectory converging to the steady state exists.\(^{14}\) We therefore have the following proposition:

**Proposition 1.** In the two-sector open economy with production externalities and with nontraded capital,

(i) if the nontraded capital good sector is labor-intensive (or capital-intensive) from both the private perspective and the social perspective (\( j_{11} j_{22} < 0 \)), the transitional dynamics for \( k_t \) and \( p_t \) exhibits saddle-path stability;

(ii) if the nontraded capital good sector is labor-intensive from the private perspective (\( j_{22} < 0 \)), but capital-intensive from the social perspective (\( j_{11} < 0 \)), then there exist an infinite number of equilibrium paths along which \( k_t \) and \( p_t \) converge to the steady state;

(iii) if the nontraded capital good sector is capital-intensive from the private perspective (\( j_{22} > 0 \) and labor-intensive from the social perspective (\( j_{11} > 0 \)), there exists no equilibrium paths along which \( k_t \) and \( p_t \) converge to the steady state.

Note that if there are no externalities (i.e., \( a = b = 0 \)), then the two eigenvalues \( j_{11} \) and \( j_{22} \) always have opposite signs, and the system has a unique saddle path. With small capital externalities, however, there are three possibilities. In case (ii), first order and transversality conditions are not sufficient to determine a unique solution path. Indeterminacy of this sort implies that the rational expectations equilibria may involve random variables, unrelated to the economy’s fundamentals, simply because agents believe it to be so.

Moreover, the externalities required to generate indeterminacy need not be large. From the expressions for the two eigenvalues in Eqs. (28) and (29), the indeterminacy condition is given by

\[ a \beta - b \alpha < \alpha \beta_1 - \alpha_1 \beta < 0. \]  

(30)

The inequality on the right (equivalent to \( j_{22} < 0 \)) specifies that the capital good sector is labor-intensive from the private perspective, while the inequality on the left (equivalent to

\(^{14}\) See, e.g., Farmer (1999).
\( j_{11} < 0 \) means that the capital good sector is capital-intensive from the social perspective. One can see that, if \((x \beta_1 - x_1 \beta)\) is negative and \textit{arbitrarily} small, the degree of capital externalities (measured by the values for \(a\) and \(b\)) needed for indeterminacy can also be \textit{arbitrarily} small. Note that the indeterminacy condition in Eq. (30) does place restrictions on the relative magnitudes of \(a\) and \(b\). For instance, indeterminacy cannot happen when \(b = 0\) and \(a \geq 0\), but can occur when \(a = 0\) and \(b > 0\). A simple example of indeterminacy is

\[
\begin{align*}
    \alpha &= 0.66, & \alpha_1 &= 0.34, & a &= 0.00; \\
    \beta &= 0.65, & \beta_1 &= 0.30, & b &= 0.05.
\end{align*}
\]

To understand the source of indeterminacy, note that the eigenvalues can be written as

\[
j_{11} = -p_t \frac{d(\frac{\mu_t}{\delta p_t})}{dp_t},
\]

and

\[
j_{22} = \frac{\partial y_{2t}}{\partial k_t} - \delta,
\]

all evaluated at the steady state, while Eq. (12) implies that

\[
\frac{\dot{p}_t}{p_t} + \frac{\mu_t}{\delta p_t} = \rho + \delta.
\]

Recall that \(\mu_t\) is the rental price of capital. Hence, \(j_{11}\) depends on the factor intensity difference reflecting the Stolper–Samuelson effect, while \(j_{22}\) relates to the factor intensity difference reflecting the Rybczynski effect.\(^{15}\) Without externalities, the two roots are of opposite signs, and the economy exhibits (unique) saddle-path stability.

What is the intuition? Assume condition (ii) in Proposition 1 is satisfied. Starting from an arbitrary equilibrium, consider an increase in the rate of investment above the initial equilibrium level, induced by an increase in \(p_t\). Since the capital good is labor-intensive from the private perspective, a rise in the capital stock decreases output at constant prices through the Rybczynski effect [see Eq. (32)]. This keeps output of capital goods from exploding. The Stolper–Samuelson effect, on the other hand, operates through social factor intensities [see Eq. (31)]. If the capital good is capital-intensive from the social perspective, the initial rise in \(p_t\) causes an increase in the returns to capital and requires a price decline to maintain the overall returns to capital equal to the world interest rate or discount rate [as required by Eq. (33)]. This offsets the initial rise in \(p_t\) and it causes it to reverse direction and move toward the steady state. Thus, indeterminacy happens because the small market imperfections break the duality between the Rybczynski and Stolper–Samuelson effects.

\(^{15}\) The Stolper–Samuelson theorem states that if, say, the price of capital-intensive good rises, the rental price of capital will rise, and the price of the other factor will fall. The Rybczynski theorem, on the other hand, states that an increase in, say, endowment of capital (holding output prices constant) will increase the output of the capital-intensive sector and decrease the output of the labor-intensive sector.
The key point of this paper is that the condition necessary for indeterminacy to occur is very different in the open and closed economies: condition (ii) in Proposition 1 is independent of the curvature of the utility function. By contrast, the two-sector closed-economy indeterminacy result requires the extreme assumption of linear or close-to-linear utility.\footnote{See Benhabib and Nishimura (1998), Section 2. Due to the implausibility of linear utility, Benhabib and Nishimura (1998) resort to a three-sector, even more complicated model.}

The intuition for this difference between the closed and open economies is straightforward. In the closed economy, if the agent wants to invest more, he must first curtail consumption. With enough curvature, the desire to smooth consumption may prevail over all other effects and make the decision to jump to another path infeasible. But in the open economy, the curvature of the utility function does not affect the investment decision since the agent can always borrow from the outside world to finance the additional investment. That is why the dynamics of the open economy depends only on technologies, and so does the indeterminacy condition.

Note finally that if the stock of capital and the price are indeterminate during the transition, so are the other variables of the system. In particular, the current account and the capital flows that finance it are also indeterminate. This creates the potential for large swings in several key variables governed only by self-fulfilling expectations.

### 3. Extending the basic model

In this section, we extend the model in two directions. In Section 3.1, we add traded capital. This is a necessary and realistic extension because many capital goods, such as equipment, are tradable. Including traded capital goods does not alter our main result, and a similar condition to that obtained in Section 2 on the sectoral factor intensities is all that is needed for indeterminacy. In Section 3.2, we demonstrate that externalities are not necessary to obtain the indeterminacy result; other market distortions like factor taxation have the same effect.

#### 3.1. The two-sector open economy with traded and nontraded capital

Assume that the traded good sector now produces both the traded consumption good \((c_t)\) and the traded capital good \(e_t\), such as equipment. Moreover, the nontraded goods sector produces the nontraded capital \(k_t\), such as structures. Production in the two sectors thus uses three factors, i.e., traded capital, nontraded capital and labor. We demonstrate below that the determinacy conditions for this extended model depend only on the sectoral factor intensities of labor relative to nontraded capital, just as in Section 2.

The agent maximizes

\[
0 \int_0^{\infty} u(c_t)e^{-\rho t}dt
\]
by choosing consumption \( c_t \), labor allocation decisions \((l_{1t}, l_{2t})\), capital allocation decisions \((e_{1t}, e_{2t}, k_{1t}, k_{2t})\), rates of investment \((i_{et}, i_{kt})\) and the rate of accumulation of bonds \((d_{\dot{t}})\), subject to

\[
\dot{d}_{\dot{t}} = rd_t + y_{1t} + p_t y_{2t} - c_t - i_{et} - p_t i_{kt},
\]

\[
y_{1t} = l_{1t}^a k_{1t}^{a1} e_{2t}^{a2} k_{1t}^{b1} e_{1t}^{b2}, \quad y_{2t} = l_{2t}^b k_{2t}^{b1} e_{2t}^{b2} k_{1t}^{h1} e_{2t}^{b2},
\]

\[
\dot{e}_t = i_{et} - \delta_e e_t, \quad \dot{k}_t = i_{kt} - \delta_k k_t,
\]

\[
e_{1t} + e_{2t} = e_t, \quad k_{1t} + k_{2t} = k_t, \quad l_{1t} + l_{2t} = 1,
\]

where \((x + x_1 + x_2 + a_1 + a_2) = (\beta + \beta_1 + \beta_2 + b_1 + b_2) = 1\) and \(\delta_e\) and \(\delta_k\) are the depreciation rates for traded and nontraded capital, respectively. Here, assume that there are externalities in both traded and nontraded capital.

We solve the agent’s problem in Appendix A. Consumption is again smoothed. Dynamics for the system boil down to two differential equations

\[
\dot{p}_t = p_t [r + \delta_k - \beta_1 h(p_t)],
\]

\[
\dot{k}_t = \left[\frac{x \beta_1 h(p_t)}{x \beta_1 - x_1 \beta} - \delta\right] k_t - \frac{x_1 \beta h(p_t)}{(x \beta_1 - x_1 \beta) h(p_t)},
\]

where \(h(p_t) = \Delta p_t \beta_1 \frac{a_1 + a_1}{\beta_2 (a_1 + a_1) - \beta_1 (a_1 + a_1)}\), \(\Delta\) is a positive parameter, and the function \(v(p_t)\) is defined in Appendix A [see Eq. (A.14)]. The Jacobian matrix of the linearized system of Eqs. (38) and (39) has the same property as the matrix in Eq. (27) in Section 2 in that its eigenvalues, denoted by \(\gamma_1\) and \(\gamma_2\), have the same signs as \([\beta(x_1 + a_1) - x(\beta_1 + b_1)]\) and \((x \beta_1 - x_1 \beta)\), respectively. We thus have the following proposition:

**Proposition 2.** In the two-sector open economy with production externalities in both traded and nontraded capital,

(i) if the nontraded capital good sector is labor-intensive (or capital-intensive) from both the private perspective and the social perspective \((\gamma_1, \gamma_2 < 0)\), the transitional dynamics for \(k_t\) and \(p_t\) exhibit saddle-path stability;

(ii) if the nontraded capital good sector is labor-intensive from the private perspective \((\gamma_2 < 0)\), but capital-intensive from the social perspective \((\gamma_1 < 0)\), then there exist an infinite number of equilibrium paths along which \(k_t\) and \(p_t\) converge to the steady state;

(iii) if the nontraded capital good sector is capital-intensive from the private perspective \((\gamma_2 > 0)\) and labor-intensive from the social perspective \((\gamma_1 > 0)\), there exist no equilibrium paths along which \(k_t\) and \(p_t\) converge to the steady state.
3.2. Factor taxation and indeterminacy

Velasco (1996) and Schmitt-Grohé and Uribe (1997) show that certain kinds of taxation can induce “fiscal increasing returns” and hence indeterminacy (and even multiple steady states) in dynamic models. In what follows, there are no such “fiscal increasing returns,” but distortionary factor taxation nonetheless causes indeterminacy. In order to simplify the exposition, we use the simpler framework of Section 2 with only nontraded capital. It would be straightforward to extend the analysis to the case with both traded and nontraded capital, as in Section 3.1.

Assume that factor taxation consists of fixed tax rates $s_1$ and $s_2$ on the earnings from nontraded capital inputs used in the two sectors, $k_{1t}$ and $k_{2t}$, respectively. These two tax rates are assumed generally not to be equal. The production functions for the two sectors are constant returns-to-scale without externality components, given by

$$y_{1t} = l_{1t}^a k_{11t}^{1-a}, \quad y_{2t} = l_{2t}^b k_{22t}^{1-b}.$$  \(40\)

In order to abstract from issues related to the choice of public spending and to facilitate comparison with the results of Section 2, assume that the government transfers the revenue from factor taxes to the private agent in lump-sum fashion. The agent’s budget constraint is

$$d_t = w_{1t}l_{1t} + (1 - \tau_1)r_{k_1}k_{11t} + p_1[w_{2t}l_{2t} + (1 - \tau_2)r_{k_2}k_{22t}] + rd_t - c_t - p_it_t + s_t,$$  \(41\)

where $s_t$ is per capita lump-sum transfers, equal to the government’s receipts from the taxation on capital inputs, i.e., $s_t = \tau_1r_{k_1}k_{11t} + \tau_2r_{k_2}k_{22t}$. From the firms’ profit maximization problem, $w_{1t} = (1 - \alpha)l_{1t}^a k_{11t}^{1-a}$, $r_{k_1} = (1 - \alpha)l_{1t}^a k_{11t}^{1-a}$, $w_{2t} = \beta l_{2t}^b k_{22t}^{1-b}$ and $r_{k_2} = (1 - \beta)l_{2t}^b k_{22t}^{1-b}$.

In addition to other first-order conditions similar to those in Section 2, the counterparts of Eqs. (9) and (10) in this case are

$$\alpha l_{1t}^a k_{11t}^{1-a} = \beta p_1 l_{2t}^b k_{22t}^{1-b},$$  \(42\)

and

$$(1 - \tau_1)(1 - \alpha)l_{1t}^a k_{11t}^{1-a} = (1 - \tau_2)(1 - \beta)p_1 l_{2t}^b k_{22t}^{1-b}.$$  \(43\)

Note that Eq. (42) implies that returns to labor inputs across sectors are equal, and Eq. (43) means that after-tax returns to capital inputs are the same. Interestingly, in this case,

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17 Fiscal increasing returns occur when the tax rate has to be varied in the opposite direction to the taxable factor in order to keep revenue fixed and equal to a constant exogenous level of expenditure. In our examples below, fiscal expenditures are endogenous, so that fiscal increasing returns cannot occur. Bond et al. (1996) obtain similar indeterminacy results in a closed-economy, endogenous growth model.

18 The results below can be easily extended to incorporate taxation on labor inputs. Here, we ignore it to facilitate the comparison with the framework without labor externalities in Section 2.
\[(1 - \alpha)(1 - \tau_1)\) and \((1 - \beta)(1 - \tau_2)\) play the roles of \(\alpha_1\) and \(\beta_1\) in the case with externalities in Section 2. The dynamic equations for \(p_t\) and \(k_t\) are

\[
\dot{p}_t = p_t (r + \delta - (1 - \beta)(1 - \tau_2)g(p_t)^\beta),
\]

\[
\dot{k}_t = \frac{m(p_t)}{(1 - \alpha 1 - \tau_1 - 1 - \beta 1 - \tau_2)} k_t + n(p_t),
\]

where \(g(p_t) = \xi p_t^{1/(\alpha - \beta)}\) as in Eq. (16) and \(m(p_t) > 0\) and \(n(p_t)\) are functions of \(p_t\).\(^{19}\) The two relevant eigenvalues of the Jacobian matrix of the linearized system are

\[
\chi_1 = \frac{\beta (r + \delta)}{(\beta - \alpha)},
\]

and

\[
\chi_2 = \frac{m(p^*)}{(1 - \alpha 1 - \tau_1 - 1 - \beta 1 - \tau_2)}.
\]

Thus, we have the following proposition:

**Proposition 3.** In the two-sector open economy with factor taxation,

(i) if the nontraded capital good sector is labor-intensive (or capital-intensive) from both the private perspective and the social perspective (\(\chi_1 \chi_2 < 0\)), the transitional dynamics for \(k_t\) and \(p_t\) exhibits saddle-path stability;

(ii) if the nontraded capital good sector is labor-intensive from the private perspective (\(\chi_2 < 0\)), but capital-intensive from the social perspective (\(\chi_1 < 0\)), there exist an infinite number of equilibrium paths along which \(k_t\) and \(p_t\) converge to the steady state;

(iii) if the nontraded capital good sector is capital-intensive from the private perspective (\(\chi_2 > 0\)) and labor-intensive from the social perspective (\(\chi_1 > 0\)), there exists no equilibrium paths along which \(k_t\) and \(p_t\) converge to the steady state.

The intuition behind the indeterminacy result in Proposition 3 is simple. In the previous cases, externalities break the duality between the Rybczynski and Stolper–Samuelson effects and insert a wedge between private and social returns, allowing for alternative and yet nonexplosive equilibrium paths. In this case, that wedge is created by tax rates.

Note that if \(\tau_1 = \tau_2\), then the two eigenvalues always have opposite signs and the equilibrium solution is unique. On the other hand, there are a variety of policy

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\(^{19}\) These can be obtained if we denote \((1 - \alpha)(1 - \tau_1)\) and \((1 - \beta)(1 - \tau_2)\) as \(\alpha_1\) and \(\beta_1\), respectively, just as in Section 2. From Eq. (20), we have \(m(p_t) = (z \beta_1 g(p_t)^\beta)/(z \beta_1 - \alpha_1 \beta) - \delta\) and \(n(p_t) = -((z \beta_1 g(p_t)^\beta - 1)/(z \beta_1 - \alpha_1 \beta))\).
combinations (for \( \tau_1 < \tau_2 \), and as long as \( \chi_1 < 0, \chi_2 < 0 \)) such that the two eigenvalues are both negative, and equilibrium indeterminacy arises.

4. Conclusion

We have shown that, in the presence of market imperfections caused by externalities, it is easier for indeterminacy to occur in small open economies facing a perfect world bond market than in closed economies. The required externalities can be very small. Alternatively, other market distortions like factor taxation can have the same effect. An important implication of our analysis is that economies open to international and intertemporal trade may be more vulnerable to self-fulfilling beliefs than they would be in autarchy.

One future task is to see whether plausible parametrization can generate the kinds of economic fluctuations that we observe in real-life economies. We plan to pursue this line of research in the future.

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Appendix A

In this appendix, we provide derivations for Section 3.1. The consolidated budget constraint is

\[
\dot{z}_t = rz_t + y_{1,t} + p_t y_{2,t} - c_t - e_t(r + \delta) + (\dot{p}_t - rp_t - \delta_k p_t),
\]

where \( z_t = d_t + e_t + p_t k_t \). Optimality conditions are

\[
u'(c_t) = \bar{\lambda},\]

\[
\begin{aligned}
\bar{\alpha}_1^{2 \tau - 1} k_{1, t}^{2 \tau + a_1} e_{1, t}^{a_2 + a_2} &= \beta p_t l_{2, t}^{\beta - 1} k_{2, t}^{\beta_1 + b_1} e_{2, t}^{\beta_2 + b_2}, \\
\bar{\alpha}_2 l_{1, t}^{a_1} k_{1, t}^{2 \tau + a_1} e_{1, t}^{a_2 + a_2 - 1} &= \beta_2 p_t l_{2, t}^{\beta_1 + b_1} k_{2, t} e_{2, t}^{\beta_2 + b_2 - 1}, \\
\bar{\alpha}_1 l_{1, t}^{a_1} k_{1, t}^{2 \tau + a_1} e_{1, t}^{a_2 + a_2 - 1} &= \beta_1 p_t l_{2, t}^{\beta_1 + b_1} k_{2, t} e_{2, t}^{\beta_2 + b_2 - 1}, \\
\beta_1 p_t l_{2, t}^{\beta_1 + b_1} k_{2, t} e_{2, t} &= p_t (r + \delta_k) - \dot{p}_t, \\
\beta_2 p_t l_{2, t}^{\beta_1 + b_1} e_{2, t}^{\beta_2 + b_2 - 1} &= r + \delta_c.
\end{aligned}
\]
Dividing Eq. (A.4) by Eqs. (A.3) and (A.5), respectively, we have

\[
\frac{\alpha_2}{\alpha} \frac{l_{1t}}{e_{1t}} = \frac{\beta_2}{\beta} \frac{l_{2t}}{e_{2t}}, \tag{A.8}
\]

and

\[
\frac{\alpha_2}{\alpha} \frac{k_{1t}}{e_{1t}} = \frac{\beta_2}{\beta} \frac{k_{2t}}{e_{2t}}, \tag{A.9}
\]

Eqs. (A.4) and (A.7) can be written as

\[
\frac{\alpha_2}{\alpha} \left( \frac{l_{1t}}{e_{1t}} \right)^{x} \left( \frac{k_{1t}}{e_{1t}} \right)^{x_1 + a_1} = \beta_2 p_t \left( \frac{l_{2t}}{e_{2t}} \right)^{\beta} \left( \frac{k_{2t}}{e_{2t}} \right)^{\beta_1 + b_1} \tag{A.10}
\]

and

\[
\beta_2 p_t \left( \frac{l_{2t}}{e_{2t}} \right)^{\beta} \left( \frac{k_{2t}}{e_{2t}} \right)^{\beta_1 + b_1} = r + \delta_c. \tag{A.11}
\]

In turn, Eqs. (A.8)–(A.11) can be solved jointly for \( l_{2t} / e_{2t}, k_{2t} / e_{2t} \) (and \( l_{1t} / e_{1t} \) and \( k_{1t} / e_{1t} \)) as functions of \( p_t \):

\[
l_{2t} \bigg/ e_{2t} = A_1 p_t^{\frac{-\left(x_1 + a_1\right)}{\beta (x_1 + a_1) - \alpha (\beta_1 + b_1)}}. \tag{A.12}
\]

and

\[
k_{2t} \bigg/ e_{2t} = A_2 p_t^{\frac{-\left(x_1 + a_1\right)}{\beta (x_1 + a_1) - \alpha (\beta_1 + b_1)}}, \tag{A.13}
\]

where \( A_1 \) and \( A_2 \) are positive constants. Substituting Eqs. (A.12) and (A.13) into Eq. (A.6) yields Eq. (38) in Section 3.1, which is similar to Eq. (19) in Section 2.

From Eqs. (A.12) and (A.13), we have

\[
l_{2t} \bigg/ k_{2t} = A_1 \frac{A_1 p_t^{\frac{-\left(x_1 + a_1\right)}{\beta (x_1 + a_1) - \alpha (\beta_1 + b_1)}}}{A_2} = v(p_t). \tag{A.14}
\]

Dividing Eq. (A.8) by Eq. (A.9) gives

\[
\frac{\alpha_1}{\alpha} \frac{l_{1t}}{k_{1t}} = \frac{\beta_1}{\beta} \frac{l_{2t}}{k_{2t}}. \tag{A.15}
\]

Following the same steps as in Section 2 to solve for \( k_{2t} \), and then using the market-clearing condition for the nontraded capital good, one can derive Eq. (39) in the text.
References