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Multiple transitional growth paths in endogenously growing open economies

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Abstract

This paper extends research in indeterminacy literature to a small open economy, two-sector endogenous growth model. It shows that a continuum of equilibria exist in two situations, (i) the production functions exhibit social constant returns to scale with very small or even negligible externalities; and (ii) the production functions are standard constant returns to scale without externalities but the government implements asymmetric tax policies across sectors.

JEL classification: F4; O4

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1. Introduction

Despite the explosion of research in modern growth theory, many important questions remain unsettled. Among them is how to explain the different growth rates across countries. Recent research in literature on indeterminacy uses closed-economy endogenous growth models with indeterminate equilibria to give a potential explanation for different growth rates of countries that start out with similar endowments and wealth levels. This stands as an alternative to theories that rely exclusively on fundamentals and/or initial conditions.[1]

The impetus for this line of research comes from the realization that indeterminacy can easily occur in endogenous growth models (e.g., [12,15]), augmented to include

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[1] See, for example, [2,4,8,18].

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some market imperfections with external effects or with monopolistic competition. While early models in the literature rely on relatively large increasing returns and high markups to generate indeterminacy, recent work demonstrates that only small externalities and market imperfections are required.[2]

This paper extends research on indeterminacy to a small open-economy endogenous growth model, and investigates how indeterminacy is relevant in such an environment. This is a realistic and necessary extension because most economies in the world are not closed, but small and open. It shows that, for the small open economy with unbounded growth, indeterminacy may occur under empirically plausible conditions and for technologies that exhibit social constant returns to scale with small or even negligible externalities.

In related work, to my knowledge, Lahiri [11] is the first paper on indeterminacy in a small open-economy, endogenous growth model. However, his model is extremely simple and relies on very large increasing returns (at a magnitude of 1.5) to generate indeterminacy—a feature that may limit the model's empirical plausibility. In addition, his results are driven by the assumption that consumption does not grow and contrast to the model with unbounded growth considered here. Weder [17] and Meng and Velasco [13] show that opening up closed economies to international trade and capital mobility may substantially increase the range of parameter values needed to generate indeterminacy, but those models are about an economy's short-run dynamics and fluctuations and are incapable of addressing issues pertaining to the determinants of long-run growth.

The next section of this paper presents a two-sector open-economy endogenous growth model with traded and nontraded capital and with production externalities,[3] and then characterizes conditions under which indeterminacy arises. Intuitive implications of the indeterminacy conditions are also presented in this section. Section 3 shows that externalities are not necessary to obtain the indeterminacy result. Other distortions like factor taxations that introduce a wedge between private and social returns have the same effects. In particular, if the production functions are standard constant returns to scale without externalities but the government implements asymmetric tax policies across sectors, indeterminacy remains likely.

2. An open-economy model of endogenous growth

The economy is inhabited by an infinitely lived representative agent. The agent accumulates two types of capital for rental at the competitively determined rental rate. The first is traded capital, k, and the second is nontraded capital, h. For expositional simplicity neither of these capital goods is subject to depreciation. In this section, we assume that there is no government.

[2] For recent theoretical models, see [3,14]. Recent empirical work such as Basu and Fernald [1] and Burnside et al. [7] finds little evidence of large increasing returns or market imperfections, which is in contrast to early findings by Hall [10] and others.

[3] Turnovsky [16] presents such a model without production externalities.

These two forms of capital are used by the agent to produce a tradable good, y_{T}, taken to be the numeraire, and a nontradable good, y_{N}. Following Benhabib et al. [3], we assume Cobb–Douglas technologies which are constant returns to scale from the social perspective but decreasing returns to scale from the private perspective due to factor-generated externalities, that is,

$$y_{\mathrm{T}} = k_{\mathrm{T}}^{\alpha_0} h_{\mathrm{T}}^{\alpha_1} \overline{k_{\mathrm{T}}^{a_0} h_{\mathrm{T}}^{a_1}}, \quad y_{\mathrm{N}} = k_{\mathrm{N}}^{\beta_0} h_{\mathrm{N}}^{\beta_1} \overline{k_{\mathrm{N}}^{b_0} h_{\mathrm{N}}^{b_1}}, \tag{1}$$

where $\alpha_0 + \alpha_1 + a_0 + a_1 = \beta_0 + \beta_1 + b_0 + b_1 = 1$, and

$$k_{\mathrm{T}} + k_{\mathrm{N}} = k, \quad h_{\mathrm{T}} + h_{\mathrm{N}} = h. \tag{2}$$

Assume that the accumulation of traded capital

$$\dot{k} = I \tag{3}$$

involves adjustment costs represented by

$$I\left(1 + \frac{\xi}{2}\frac{I}{k}\right), \quad \xi > 0, \tag{4}$$

where the linear homogeneity of the cost function is standard, and necessary if a steady-state equilibrium having ongoing growth is to be sustained. As will be clear below, the assumption of adjustment costs for traded capital accumulation is the key feature of the model and necessary to give rise to nondegenerate dynamics.

In addition to the two types of capital, the agent accumulates net foreign bonds, b, that pay an exogenously given world interest, r. Thus the agent's budget constraint is given by

$$\dot{b} = rb + y_{\mathrm{T}} + py_{\mathrm{N}} - c_{\mathrm{T}} - pc_{\mathrm{N}} - I\left(1 + \frac{\xi}{2}\frac{I}{k}\right) - p\dot{h}. \tag{5}$$

The agent's problem is to choose $c_{\mathrm{T}}, c_{\mathrm{N}}$, $k_{\mathrm{T}}, k_{\mathrm{N}}, h_{\mathrm{T}}, h_{\mathrm{N}}$, I and $\dot{h}$ to maximize the following intertemporal utility function:

$$U = \int_0^\infty \frac{1}{\gamma}\left(c_{\mathrm{T}}^{\theta} c_{\mathrm{N}}^{1-\theta}\right)^{\gamma} e^{-\rho t}\, dt \tag{6}$$

subject to (1)–(5) and the initial stocks of assets k_0, h_0 and b_0.

The Hamiltonian for this optimization problem is

$$\begin{aligned}
\mathscr{H} = {} & \frac{1}{\gamma}\left(c_{\mathrm{T}}^{\theta} c_{\mathrm{N}}^{1-\theta}\right)^{\gamma} e^{-\rho t} + (\lambda q) e^{-\rho t}[I - \dot{k}] \\
& + \lambda e^{-\rho t}\Big[rb + k_{\mathrm{T}}^{\alpha_0} h_{\mathrm{T}}^{\alpha_1} \overline{k_{\mathrm{T}}^{a_0} h_{\mathrm{T}}^{a_1}} + p k_{\mathrm{N}}^{\beta_0} h_{\mathrm{N}}^{\beta_1} \overline{k_{\mathrm{N}}^{b_0} h_{\mathrm{N}}^{b_1}} - c_{\mathrm{T}} \\
& - pc_{\mathrm{N}} - I\left(1 + \frac{\xi}{2}\frac{I}{k}\right) - p\dot{h} - \dot{b}\Big].
\end{aligned}$$

The first-order conditions with respect to $c_{\mathrm{T}}, c_{\mathrm{N}}$, $k_{\mathrm{T}}, k_{\mathrm{N}}, h_{\mathrm{T}}, h_{\mathrm{N}}$, I are given by

$$\theta c_{\mathrm{T}}^{\theta\gamma - 1} c_{\mathrm{N}}^{\gamma(1-\theta)} = \lambda, \tag{7}$$

$$(1-\theta) c_{\mathrm{T}}^{\theta\gamma} c_{\mathrm{N}}^{\gamma(1-\theta)-1} = \lambda p, \tag{8}$$

$$\alpha_0 k_{\mathrm{T}}^{\alpha-1} h_{\mathrm{T}}^{1-\alpha} = p\beta_0 k_{\mathrm{N}}^{\beta-1} h_{\mathrm{N}}^{1-\beta} \equiv r_k, \tag{9}$$

$$\alpha_1 k_{\mathrm{T}}^{\alpha} h_{\mathrm{T}}^{-\alpha} = p\beta_1 k_{\mathrm{N}}^{\beta} h_{\mathrm{N}}^{-\beta} \equiv r_h, \tag{10}$$

$$1 + \xi \frac{I}{k} = q, \tag{11}$$

where to simplify exposition the following notations are introduced:

$$\alpha \equiv \alpha_0 + a_0, \quad \beta \equiv \beta_0 + b_0. \tag{12}$$

Eq. (11) can be solved to yield the following expression for the rate of accumulation of traded capital:

$$\frac{I}{k} = \frac{\dot{k}}{k} = \frac{q-1}{\xi} \equiv \phi(t). \tag{13}$$

The standard optimality conditions with respect to traded bond b, and the two forms of capital, k and h, lead to the arbitrage conditions

$$\rho - \frac{\dot{\lambda}}{\lambda} = r, \tag{14}$$

$$\frac{r_k}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2\xi q} = r, \tag{15}$$

$$\frac{r_h}{p} + \frac{\dot{p}}{p} = r. \tag{16}$$

Eq. (14) equates the marginal returns on consumption to the fixed rate of return on holding a foreign bond. Since both ρ and r are constants, it implies a constant growth rate of the marginal utility λ. Note that in contrast to the usual Ramsey model of the small open economy, where $\rho = r$ is required for a well-defined steady-state equilibrium to exist, this equality is not necessary for a well-defined equilibrium to obtain in a growing economy. However, some other assumptions are required so that the economy will satisfy its intertemporal budget constraint and thus remain intertemporally solvent (see below). Eq. (15) equates the rate of return on traded bond to the rate of return on capital taking into account of the costs of capital installation. Eq. (16) equates the total rate of return on nontraded capital to the rate of return on the traded bond.

Finally, the following transversality conditions must be met:

$$\lim_{t\to\infty} \lambda b e^{-\rho t} = 0, \quad \lim_{t\to\infty} (\lambda q) k e^{-\rho t} = 0, \quad \lim_{t\to\infty} \lambda p h e^{-\rho t} = 0. \tag{17}$$

2.1. *Equilibrium dynamics*

Define aggregate consumption c as $c \equiv c_{\mathrm{T}} + pc_{\mathrm{N}}$, which, along with the first-order conditions (7), (8) and (14), leads to the following relations:

$$c_{\mathrm{T}} = \theta c, \quad pc_{\mathrm{N}} = (1-\theta)c, \tag{18}$$

$$\frac{\dot{c}}{c} = \frac{r - \rho - \gamma(1-\theta)(\dot{p}/p)}{1-\gamma} \equiv \psi(t). \tag{19}$$

In addition, it is straightforward to derive the following expressions for r_k and r_h as functions of the relative price p:

$$r_k(p) = \alpha_0 \delta^{\alpha-1} p^{\frac{\alpha-1}{\alpha-\beta}},$$

$$r_h(p) = \alpha_1 \delta^{\alpha} p^{\frac{\alpha}{\alpha-\beta}}, \quad \text{where } \delta = \left[\left(\frac{\beta_0}{\alpha_0}\right)^{\beta}\left(\frac{\beta_1}{\alpha_1}\right)^{1-\beta}\right]^{(1/(\alpha-\beta))}. \tag{20}$$

By substitution, the following dynamic equations for the relative price, p, and the price of installed traded capital, q, are obtained

$$\dot{p} = rp - \alpha_1 \delta^{\alpha} p^{\frac{\alpha}{\alpha-\beta}}, \tag{21}$$

$$\dot{q} = rq - \frac{(q-1)^2}{2\xi} - \alpha_0 \delta^{\alpha-1} p^{\frac{\alpha-1}{\alpha-\beta}}. \tag{22}$$

The critical determinant of the growth of traded capital is the market price of installed capital q (see (13)). In order for the traded capital ultimately to follow a path of steady growth or decline, the stationary solutions to this pair of equations, attained when $\dot{p} = \dot{q} = 0$, must have at least one real solution. Thus the steady-state relative price of nontraded goods $\tilde{p}$ must be either 0, which is ruled out, or

$$\tilde{p} = [r/\alpha_1 \delta^{\alpha}]^{(\frac{\alpha-\beta}{\beta})}. \tag{23}$$

The corresponding steady-state value of q, denoted by $\tilde{q}$, is the solution to the quadratic equation

$$r_k(\tilde{p}) + \frac{(\tilde{q}-1)^2}{2\xi} = r\tilde{q}. \tag{24}$$

A necessary and sufficient condition for the stock of traded capital ultimately to converge to a steady growth path is that this equation should have real roots, and this will be so if and only if

$$r_k(\tilde{p}) \leqslant r\left[1 + \frac{\xi r}{2}\right]. \tag{25}$$

Notice that the smaller the adjustment cost ξ, the smaller must the marginal physical product of traded capital be, in order for a balanced growth path for capital to exist. This is because there is a trade-off between the first and the third components of the rates of return to traded capital given by the left-hand side of (15). The smaller the adjustment cost ξ, the greater the returns to capital due to valuation differences between installed capital and the embodied resources, and the greater the incentives to transform new output to capital. If for a given ξ, $r_k(\tilde{p})$ is sufficiently large to reverse (25), the returns to traded capital dominate the returns to bonds (and also the nontraded capital), irrespective of the price of installed capital, so that no long-run balanced equilibrium can exist.

The roots for (24) are denoted by $\tilde{q}_1 = (1+\xi r) - \sqrt{(1+\xi r)^2 - (1+2\xi r_k(\tilde{p}))}$ (smaller) and $\tilde{q}_2 = (1+\xi r) + \sqrt{(1+\xi r)^2 - (1+2\xi r_k(\tilde{p}))}$ (larger). In the case that (25) holds, the behavior of prices can be summarized by the following lemma:

Lemma. (i) *If $\beta > \alpha$, the only solutions for p and q which are consistent with the transversality condition on traded capital are the steady state $p = \tilde{p}$, and $q = \tilde{q}_1$, the (unstable) steady-state solution given by the smaller root. In this case there are no transitional dynamics in either the relative price of nontraded capital goods or the market price of capital.*

(ii) *If $\alpha > \beta$, the only solutions for p and q which are consistent with the transversality condition on traded capital are that p and q lie on the stable saddle path, ultimately converging to $p = \tilde{p}$, and $q = \tilde{q}_1$, the steady-state solution given by the smaller root.*

Many would argue that (ii) is more plausible of these two cases; i.e., the tradable sector is relatively intensive in tradable capital (from social perspective).[4] As we will show below, it is right in this case that indeterminacy may occur with small externalities in productions.

Notice both prices are forward-looking variables—both of which may respond instantaneously to new information. In the case that $\alpha > \beta$, their respective initial values $p(0)$ and $q(0)$ will be at the stable saddle path. But what other conditions can pin down $p(0)$ and $q(0)$? As is shown in [16], in the absence of any externalities or market imperfections, they are uniquely determined by one more condition: the transversality condition on nontraded wealth (see footnote 6 below for his argument). We will show below, nevertheless, that with small externalities in the technologies, the transversality condition on nontraded wealth still may not determine the initial prices. Thus the prices and growth rates or levels of other variables are all indeterminate during transition to the steady-state growth path.

Define aggregate wealth in terms of the traded good as

$$w = qk + ph + b. \tag{26}$$

The following relationship describing the rate of aggregate wealth accumulation holds:

$$\dot{w}(t) = rw(t) - c(t). \tag{27}$$

From (19), it can be obtained that $c(t) = c(0)e^{\int_0^t \psi(s)\,ds}$. In particular, at the steady-state growth path (i.e., $\dot{p} = 0, \dot{q} = 0$)

$$\frac{\dot{c}}{c} = \frac{r-\rho}{1-\gamma} \equiv \tilde{\psi}. \tag{28}$$

[4] See, e.g., [6,9]. Theoretically, however,case (i) cannot be ruled out.

Solving (27) and invoking the transversality condition yields

$$\frac{c}{w} = \frac{1}{\int_t^{\infty} e^{\int_t^{\tau}[r-\psi(s)]\,ds}\,d\tau}. \tag{29}$$

Using (27) and the solution for $c(t)$, the equilibrium rate of growth of aggregate wealth can be expressed as

$$\frac{\dot{w}}{w} = \frac{\dot{c}}{c} + \left\{(r-\psi) - \frac{c}{w}\right\}.$$

Therefore, both w and c have transitional dynamic time paths. At the steady-state growth path ($\dot{p} = 0$), we have

$$\frac{c}{w} = r - \tilde{\psi},$$

$$\frac{\dot{w}}{w} = \frac{\dot{c}}{c} = \frac{r-\rho}{1-\gamma} \equiv \tilde{\psi}.$$

Thus for a positive constant aggregate consumption/wealth ratio, the following condition should be imposed:[5]

$$r > \tilde{\psi}. \tag{30}$$

This restriction, along with (29), implies that the transversality condition for aggregate wealth is satisfied (noting $\lambda(t) = \lambda(0)e^{(\rho-r)t}$)

$$\lim_{t-0} \lambda w e^{-\rho t} = 0.$$

Since the transversality condition holds for traded capital k, we need to check whether and under what conditions the transversality conditions for h and b are satisfied. We now examine the transversality condition for nontraded capital h. It is noted that when the transversality condition for h is satisfied, the other transversality condition for net foreign bonds b will also be met since $b = w - qk - ph$.

From the appendix, a linear approximation to the solution for the time path of $p(t)h(t)$, valid around the steady-state growth path and when the solutions for asset prices are near their respective steady-state levels, is given by

$$p(t)h(t) = A_1 e^{-r(\frac{\alpha_0}{\beta_0\alpha_1-\beta_1\alpha_0})t} + A_2 e^{\tilde{\phi}t} + A_3 e^{\tilde{\psi}t}, \tag{31}$$

where

$$\begin{aligned} A_1 =& p(0)h_0 - \frac{\alpha_1\tilde{r}_k}{(\beta_0\alpha_1-\beta_1\alpha_0)[\tilde{\phi} + r(\frac{\alpha_0}{(\beta_0\alpha_1-\beta_1\alpha_0)})]} \\ &+ \frac{(1-\theta)(r-\tilde{\psi})}{[\tilde{\psi} + r(\frac{\alpha_0}{(\beta_0\alpha_1-\beta_1\alpha_0)})]}[q(0)k_0 + p(0)h_0 + b_0], \end{aligned} \tag{32}$$

[5] Or equivalently, $(\rho - r\gamma)/(1-\gamma) > 0$, which imposes a minimal condition for these parameters. For example, $\gamma = 0$ (i.e., the utility function is logarithmic) satisfies this condition.

$\tilde{\phi}$ is the steady-state growth rate of traded capital, i.e., $\frac{\dot{k}}{k} = \frac{\tilde{q}-1}{\xi} = \tilde{\phi}$, and the values for A_2 and A_3 are listed in the appendix. Note that the transversality condition for nontraded capital in (17) can be written as

$$\lim_{t \to \infty} phe^{-rt} = 0. \tag{33}$$

Applying (33) to the three exponential terms in (31), the following conditions must hold: (i) $r(\frac{\alpha_0}{\beta_0\alpha_1 - \beta_1\alpha_0}) + r > 0$; (ii) $r > \tilde{\phi}$; (iii) $r > \tilde{\psi}$. Condition (ii) is satisfied by the smaller root of q_1, while condition (iii) has been imposed by the transversality condition upon aggregate wealth and it is necessary and sufficient for the consumption/wealth ratio to be nonnegative.

For condition (i), in the case that $\alpha > \beta$, if there are no externalities, then $\alpha_0 = \alpha$, $\beta_0 = \beta$, $\alpha_1 = 1 - \alpha$, $\beta_1 = 1 - \beta$, and $r(\frac{\alpha_0}{\beta_0\alpha_1 - \beta_1\alpha_0}) + r = r(\frac{\alpha}{\beta - \alpha}) + r < 0$, condition (i) will be violated if $A_1 \neq 0$.[6]

Despite this problem for the case without externalities, condition (i) may still hold even if $A_1 \neq 0$ in the presence of externalities in production functions. This occurs when $(\beta_0\alpha_1 - \beta_1\alpha_0) > 0$ so that $r(\frac{\alpha_0}{\beta_0\alpha_1 - \beta_1\alpha_0}) + r = r(\frac{\beta_0\alpha_1 + \alpha_0(1-\beta_1)}{\beta_0\alpha_1 - \beta_1\alpha_0}) > 0$. In other words, the transversality condition for nontraded capital is satisfied for any possible initial values of the price levels along the convergent path around the steady state. We have therefore the following proposition:

Proposition 1. *In the two-sector open economy of endogenous growth with production externalities, if the traded sector is relatively intensive in traded capital from social perspective* ($\alpha > \beta$) *and relatively intensive in nontraded capital from private perspective* $((\beta_0\alpha_1 - \beta_1\alpha_0) > 0)$, *then the steady state for prices p and q is indeterminate in the sense that there are a continuum of initial values of prices, and there exist a continuum of growth rates of output, consumption and capital stocks that converge respectively to their steady-state growth paths.*

It is clear that examples satisfying the above conditions for indeterminacy can be constructed with arbitrarily small external effects. To see the intuitive implications for the indeterminacy conditions, notice that indeterminacy arises because of the presence of small external effects. The price dynamics depends on $(\alpha - \beta)$, or the social relative capital intensity. If $\alpha > \beta$, the dynamics for $p(t)$ and $q(t)$ follows a stable path. This is because when the price of traded capital intensive good rises, the rental price of the traded capital will increase, and the rental price of the other factor (nontraded capital) will fall (see (16), (21) and (15), (22)). This reflects the Stolper–

[6] In the no externality case in [16], A_1 is set to 0 to ensure that the transversality condition is still met. It is then claimed that $A_1 = 0$, combined with the stable saddle path equation for prices p and q, uniquely determines the two initial price levels $p(0)$ and $q(0)$. But this argument has one flaw. Notice that the expression for A_1 is obtained based on the assumption that the initial prices are in the small neighborhood of the steady state, but if $A_1 = 0$ and the saddle path equation are jointly solved, the solution will not be in the neighborhood if it is sufficiently small. That is, any possible initial values of the prices in the neighborhood of the steady state do not satisfy $A_1 = 0$, and the dynamic system is unstable.

Samuelson theorem in trade theory. On the other hand, if the traded good is nontraded capital intensive from the private perspective, an increase in nontraded capital decreases the nontraded good output at constant prices through the Rybczynski effect. This is the dominating effect on the right-hand side of (31), which keeps the nontraded capital from exploding at a faster rate than that of the foreign bonds, and ensures that the transversality condition for nontraded capital holds.

We next look at more closely how openness or international capital mobility matters for the indeterminacy conditions. If the initial prices $p(0)$, $q(0)$ and their paths are chosen, then the initial aggregate wealth and domestic consumption are determined by (26) and (29) (the latter is derived from the budget constraint). Notice, however, that from (19) the growth rate for consumption depends only on the relative price p and other parameters. This is due to the fact that the small open economy with perfect international capital mobility faces an exogenously given world interest rate r, and hence the marginal utility is constant. Suppose initially the economy stays at the steady-state growth path, the agent decides to increase the growth rate of traded capital, then from (13) the price of installed traded capital q jumps up to reflect this increase. If the factor intensity conditions hold as in Proposition 1, then q and p fall upon the stable path that converges to the steady state. The new starting prices for p and q, along with the new capital stocks, determine the new starting consumption level. The subsequent consumption time path, from (19), once again depends only on the relative price p, and is independent of the ensuing investment decisions (at the same time their transversality conditions hold). This explains why taste parameters and in particular the curvature of the utility function (the parameter γ) are not present in the indeterminacy conditions, except for (30), which imposes a weak condition on the taste parameters for the consumption/wealth ratio to be nonnegative (see footnote 5).

Finally, we have concentrated so far on the case that $\alpha > \beta$. In the case that $\alpha < \beta$ (which cannot be ruled out in theory), following the same line of reasoning, and in conjunction with the lemma (part (i)), one can show that if $(\beta_0\alpha_1 - \beta_1\alpha_0) < 0$, no equilibrium exists and the system is unstable. In the case without externalities or in general $(\beta_0\alpha_1 - \beta_1\alpha_0) > 0$ there is a unique solution since the transversality condition is automatically satisfied, in which case the only solution for the prices is the steady state.

3. Factor taxation and indeterminacy

This section shows that if the government taxes or subsidizes the traded and nontraded sectors in an asymmetric way, indeterminacy of transitional paths may still arise even without externalities in production functions. In particular, it is assumed that the tax (or subsidy) rates on factor inputs in two sectors may not be equal. This asymmetry in taxation will prove to have the same effects as the externality components in the previous section.[7]

[7] Bond et al. [5] analyze the effects of factor taxation on transitional dynamics in the context of a closed-economy endogenous growth model. The indeterminacy results obtained below are more general.

Assume factor taxation consists of four different tax rates $\tau_{k_{\rm T}}, \tau_{k_{\rm N}}, \tau_{l_{\rm T}}, \tau_{l_{\rm N}}$ on the earnings from factors employed in the two sectors (negative tax rates imply subsidy). The production functions for the two sectors are

$$y_{\rm T} = k_{\rm T}^{\alpha} h_{\rm T}^{1-\alpha}, \quad y_{\rm N} = k_{\rm N}^{\beta} h_{\rm N}^{1-\beta}, \ 0<\alpha, \ \beta<1. \tag{34}$$

In order to isolate the effects of taxation from those of the government expenditure and to facilitate comparison with the result of the previous section, we assume that the revenue from factor taxes is used to finance lump-sum transfer payments.

The agent's intertemporal budget constraint is now given by

$$\begin{aligned}\dot{b} = {} & (1-\tau_{h_{\rm T}}) w_{h_{\rm T}} h_{\rm T} + (1-\tau_{k_{\rm T}}) r_{k_{\rm T}} k_{\rm T} + p[(1-\tau_{h_{\rm N}}) w_{h_{\rm N}} h_{\rm N} \\ & + (1-\tau_{k_{\rm N}}) r_{k_{\rm N}} k_{\rm N}] + rb - c_{\rm T} - pc_{\rm N} - I\left(1+\frac{\xi}{2}\frac{I}{k}\right) - p\dot{h} + \Upsilon,\end{aligned} \tag{35}$$

where $\Upsilon = \tau_{h_{\rm T}} w_{h_{\rm T}} h_{\rm T} + \tau_{k_{\rm T}} r_{k_{\rm T}} k_{\rm T} + p[\tau_{h_{\rm N}} w_{h_{\rm N}} h_{\rm N} + \tau_{k_{\rm N}} r_{k_{\rm N}} k_{\rm N}]$, and the pre-tax earnings are given by $r_{k_{\rm T}} = \alpha k_{\rm T}^{\alpha-1} h_{\rm T}^{1-\alpha}$, $w_{h_{\rm T}} = (1-\alpha) k_{\rm T}^{\alpha} h_{\rm T}^{-\alpha}$, $r_{k_{\rm N}} = \beta k_{\rm N}^{\beta-1} h_{\rm N}^{1-\beta}$, $w_{k_{\rm N}} = (1-\beta) k_{\rm N}^{\beta} h_{\rm N}^{-\beta}$.

The optimization problem is to choose $c_{\rm T}, c_{\rm N}, k_{\rm T}, k_{\rm N}, h_{\rm T}, h_{\rm N}, I$ and $\dot{h}$ to maximize (6), subject to (2), (3), (34) and (35). By introducing the notations, $\alpha_0 = (1-\tau_{k_{\rm T}})\alpha$, $\alpha_1 = (1-\tau_{h_{\rm T}})(1-\alpha)$, $\beta_0 = (1-\tau_{k_{\rm N}})\beta$, $\beta_1 = (1-\tau_{h_{\rm N}})(1-\beta)$, and using the same notations for other counterpart variables used in the previous section, it is easy to see that the optimality conditions are identical in the two cases. The price dynamics remain the same under both sectorial relative capital intensity conditions. In particular, the conditions for indeterminacy do not change, and indeterminacy arises when $\alpha>\beta$ and $(\beta_0\alpha_1 - \beta_1\alpha_0)>0$. The latter inequality implies

$$\frac{\beta}{1-\beta}\frac{(1-\tau_{k_{\rm N}})}{(1-\tau_{h_{\rm N}})} > \frac{\alpha}{1-\alpha}\frac{(1-\tau_{k_{\rm T}})}{(1-\tau_{h_{\rm T}})}. \tag{36}$$

The next proposition summarizes the result:

Proposition 2. *In the two-sector open economy of endogenous growth with CRS technologies and factor taxations, if the traded sector is relatively intensive in traded capital from social perspective ($\alpha>\beta$) and relatively intensive in nontraded capital from private perspective ($\frac{\beta}{1-\beta}\frac{(1-\tau_{k_{\rm N}})}{(1-\tau_{h_{\rm N}})} > \frac{\alpha}{1-\alpha}\frac{(1-\tau_{k_{\rm T}})}{(1-\tau_{h_{\rm T}})}$), then the steady state for prices p and q is indeterminate in the sense that there are a continuum of initial values of prices, and there exist a continuum of growth rates of output, consumption and capital stocks that converge respectively to their steady-state growth paths.*

Note that if $\tau_{k_{\rm T}} = \tau_{k_{\rm N}}$, $\tau_{h_{\rm T}} = \tau_{h_{\rm N}}$, or generally $\frac{1-\tau_{k_{\rm N}}}{1-\tau_{h_{\rm N}}} = \frac{1-\tau_{k_{\rm T}}}{1-\tau_{h_{\rm T}}}$, (36) cannot hold. On the other hand, there are a variety of policy combinations such that (36) is satisfied if at least one of $\tau_{k_{\rm T}} = \tau_{k_{\rm N}}$ and $\tau_{h_{\rm T}} = \tau_{h_{\rm N}}$ does not hold (i.e., factor taxations must be asymmetric just as the externality components in the previous section). For example, in the case that $\tau_{h_{\rm T}} = \tau_{h_{\rm N}}$, (36) may be satisfied for $\tau_{k_{\rm T}} < \tau_{k_{\rm N}}$ (the magnitudes depending on the relative values of α and β). Another example is the case when $\tau_{k_T} = \tau_{k_N}$, (36) may be satisfied for $\tau_{h_T} > \tau_{h_N}$. These two examples suggest that indeterminacy of growth paths may arise when the government implements preferential policies toward either the traded or nontraded sector.

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Appendix

This appendix derives (31) in the text. By using (2), (9), (10) and (20), we obtain, $y_N = \frac{1}{p}\frac{\alpha_1 r_k k - \alpha_0 r_h h}{(\beta_0\alpha_1 - \beta_1\alpha_0)}$. Thus

$$\frac{d}{dt}(ph) = p\dot{h} + \dot{p}h = \left(r + \frac{r_h}{p}\frac{(\beta_0\alpha_1 - \beta_1\alpha_0 + \alpha_0)}{(\beta_1\alpha_0 - \beta_0\alpha_1)}\right)ph - \frac{\alpha_1 r_k k}{(\beta_1\alpha_0 - \beta_0\alpha_1)} - (1-\theta)c.$$

As $\dot{p} = p(r - \frac{r_h}{p}) = 0$ at the steady state, the above expression near the steady state can be approximated by

$$\frac{d}{dt}(ph) = r\frac{\alpha_0}{(\beta_1\alpha_0 - \beta_0\alpha_1)}ph - \frac{\alpha_1 \tilde{r}_k}{(\beta_1\alpha_0 - \beta_0\alpha_1)}k - (1-\theta)c.$$

Since $k(t) = k_0 e^{\tilde{\phi}t}$ and $c(t) = c(0)e^{\tilde{\psi}t}$ at the steady-state growth path, a linear approximation to the solution for the time path of $p(t)h(t)$, valid around the steady-state growth path and when the solutions for asset prices are near their respective steady-state levels, is given by

$$p(t)h(t) = A_1 e^{-r(\frac{\alpha_0}{\beta_0\alpha_1 - \beta_1\alpha_0})t} + A_2 e^{\tilde{\phi}t} + A_3 e^{\tilde{\psi}t},$$

where A_1 is (32) in the text, and

$$A_2 = \frac{\alpha_1\tilde{r}_k}{(\beta_0\alpha_1 - \beta_1\alpha_0)[\tilde{\phi} + r(\frac{\alpha_0}{(\beta_0\alpha_1 - \beta_1\alpha_0)})]}k_0, \quad A_3 = -\frac{(1-\theta)}{[\tilde{\psi} + r(\frac{\alpha_0}{(\beta_0\alpha_1 - \beta_1\alpha_0)})]}c(0).$$

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