Monetary policy and multiple equilibria in a cash-in-advance economy

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Received 15 October 2000; accepted 15 May 2001

Abstract

This article shows that whether active interest rate rules can generate equilibrium uniqueness or indeterminacy depends upon: (1) the magnitude of the intertemporal elasticity of substitution, and (2) the value of the steady state inflation rate. In particular, if the intertemporal elasticity is smaller than one, then an active interest rate rule renders equilibrium indeterminate when the steady state inflation rate is sufficiently high, and ensures uniqueness of equilibrium when it is sufficiently low. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Interest rate rules; Indeterminacy

JEL classification: E31; E52

1. Introduction

A well-known result in the literature on interest rate policy is that active interest rate feedback rules, that is, rules that respond to increases in inflation with a more than one-for-one increase in the nominal interest rate, are stabilizing by ensuring uniqueness of equilibrium. Some authors have recently started to challenge the early results. For example, Benhabib et al. (2001) demonstrate that depending on how money enters preference or production, active interest rate rules may destabilize the economy by generating equilibrium indeterminacy.

Results obtained by all prior authors suggest whether or not a particular interest rate rule generates equilibrium uniqueness or indeterminacy is independent of the value of the steady state inflation rate. Therefore, all these results imply that regardless of the magnitude of the steady state inflation rate,
certain policy either stabilizes or destabilizes the economy. The present paper shows that, however, whether active interest rate rules can generate uniqueness of equilibrium depends upon the magnitude of the steady state inflation rate. In particular, in a standard cash-in-advance economy with non-separable utility in consumption and leisure, if the intertemporal elasticity is smaller than one, then active interest rate rules render equilibrium indeterminate if the steady state inflation rate is sufficiently high, and ensure uniqueness of equilibrium if the steady state inflation rate is sufficiently low.

2. A cash-in-advance economy

2.1. The model

The representative household’s lifetime utility is given by:

\[ U = \int_0^\infty e^{-\rho t} u(c_t, l_t) \, dt \]  \hspace{1cm} (1)

where \( \rho > 0 \) denotes the rate of time preference, \( c_t \) consumption, \( l_t \) leisure. Assume that cash has to be held in advance of purchasing goods. The liquidity constraint that the household faces can be formalized as:

\[ M_t \equiv \alpha P_t c_t \]  \hspace{1cm} (2)

where \( M_t \) are nominal money balances held for purchases, \( P_t \) is the nominal price level, and \( \alpha \) is a positive parameter. The household also holds nominal bonds, \( B_t \), which pay the nominal interest rate \( R_t > 0 \).

The household is subject to the liquidity and budget constraints:

\[ m_t \equiv \alpha c_t \]  \hspace{1cm} (3)

\[ \dot{a}_t = (R_t - \pi_t) a_t - R_t m_t + y_t - c_t - \tau_t \]  \hspace{1cm} (4)

where \( m_t \) are real cash balances, \( \pi_t \) the inflation rate, \( \tau_t \) lump taxes, \( a_t = (M_t + B_t) / P_t \), and \( y_t \) the output flow. Substitute (3) into (4), we have:

\[ \dot{a}_t = (R_t - \pi_t) a_t - c_t (1 + \alpha R_t) + y_t - \tau_t \]  \hspace{1cm} (5)

The household is also subject to the following no-Ponzi-game condition:

\[ \lim_{t \to \infty} e^{-\int_0^t [R(s) - \pi(s)] \, ds} a_t \geq 0 \]  \hspace{1cm} (6)

The household maximizes (1) subject to (5) and (6). The optimality conditions are:

\[ u(c_t, l_t) = \lambda_t (1 + \alpha R_t) \]  \hspace{1cm} (7)
where term $(1 + \alpha R_i)$ is the usual monetary ‘wedge’ generated in cash-in-advance models, due to the fact that any form of wealth (except cash itself) has to be made liquid before goods can be consumed. This increases the effective price of consumption but not that of leisure. $w$, here is the real wage rate.

We assume the following standard utility function used extensively in the RBC literature:

$$u(c, l) = \lambda w$$

where $\lambda$ is the real wage rate.

We assume the following standard utility function used extensively in the RBC literature:

$$u(c, l) = (c^{\beta} l^{1-\beta})^{1-\sigma} / (1 - \sigma), \quad 0 < \beta < 1$$

Assume that the household is endowed with 1 unit of time and choose endogenously to have leisure and to work. To simplify the analysis, we assume the linear production function:

$$y = h_i$$

where $h_i = 1 - l_i$ is the labor supply.

### 2.2. Interest rate rules

Following Leeper (1991) and Benhabib et al. (2001), we assume that the monetary policy takes the form of an interest rate feedback rule whereby the nominal interest rate is set as an increasing function of inflation rate. Specifically:

$$R_i = \psi(\pi_i)$$

where $\psi(\cdot)$ is continuous, non-decreasing, and there exists at least one $\pi^* > -\rho$ such that $\psi(\pi^*) = \rho + \pi^*$. We say that the monetary authority implements active monetary policy if $\psi'(\pi^*) > 1$, and passive monetary policy if $\psi'(\pi^*) < 1$.

Government purchases are assumed to be zero at all times. The budget constraint of the government is given by $\dot{B}_t = R_i B_t - M_t - P_t \tau$, which can be written as:

$$\dot{a}_t = (R_i - \pi) a_t - R_i m_t - \tau$$

We assume the fiscal policy is Ricardian in the sense that (10) is always satisfied both in and off equilibrium.

### 2.3. Equilibrium and local dynamics

By substitution, the first-order conditions are:

$$\left(c^{\beta} l^{1-\beta}\right)^{-\sigma} \beta c^{\beta-1} l^{1-\beta} = \lambda (1 + \alpha \psi(\pi_i))$$

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2The major results obtained below do not change if we instead use the production function $y_i = h_i^\eta$ ($0 < \eta < 1$).
In equilibrium, the goods market clear:
\[ c_i = h_i = 1 - l_i \]  
(17)

From (14), (15) and (17), the following equation can be derived:
\[ \frac{\beta}{(1 - \beta)} \frac{l_i}{(1 - l_i)} = 1 + \alpha \psi(\pi_i) \]  
(18)

Since \( \psi(\pi_i) \) is an increasing function of \( \pi_i \), from (18), \( l_i \) is also an increasing function of \( \pi_i \).

Therefore:
\[ l_i = l(\pi_i), \quad l'(\pi_i) > 0 \]  
(19)

From (17) \( c_i \) is a decreasing function of inflation rate \( \pi_i \)
\[ c_i = c(\pi_i), \quad c'(\pi_i) < 0 \]  
(20)

Intuition for both (19) and (20) is straightforward. When inflation rate is high, people have less incentive to work and hence consume more leisure. They also consume less of regular goods when the cost of purchasing is high.

From (15) and (17),
\[ \lambda_i = (1 - \beta)(1 - l_i)^{1 - \sigma} \frac{l_i^{(1 - \beta)} + \beta}{l_i^{(1 - \beta)}} \]  
(21)

Since \( l_i \) is a function of \( \pi_i \), from (21) \( \lambda_i \) is also a function of \( \pi_i \). (16) can be written as:
\[ \dot{\pi}_i = \frac{\lambda(\pi_i)(\rho + \pi_i - \psi(\pi_i))}{\lambda'(\pi_i)} \]  
(22)

Linearize (21) at the steady state \( \pi^* \).
\[ \ddot{\pi}_i = \frac{\Lambda(\pi^*)(1 - \psi'(\pi^*))}{\Lambda'(\pi^*)}(\pi_i - \pi^*) \]  
(23)

The dynamics of the above differential equation hinges on the sign of
\[ \Delta = \frac{\Lambda(\pi^*)(1 - \psi'(\pi^*))}{\Lambda'(\pi^*)} \]
In particular, if \( \Delta > 0 \) the unique solution is the steady state \( \pi_i = \pi^* \). If \( \Delta < 0 \), on the other hand, there are a continuum of equilibrium trajectories that converge to the steady state, and hence equilibrium indeterminacy arises. In order to see the connection between the stance of interest rate policies and equilibrium, we consider next two cases separately.

**Case 1.** \( \sigma \approx 1 \) \( (c_i \) and \( l_i \) are separable when \( \sigma = 1 \)), from (21), the right-hand side is a decreasing
function of $l_t$, and hence a decreasing function of $\pi_t$. Therefore $\lambda'(\pi_t) < 0$. We have the following proposition.

**Proposition 1.** If the intertemporal elasticity of substitution (the inverse of $\sigma$) is not less than one, then for active interest rate rules the only equilibrium is the steady state.

**Case 2.** $\sigma > 1$, (21) can be written as:

$$\lambda_t = \frac{(1 - \beta)}{(1 - l_t)\beta(\sigma - 1)}l^{\sigma(1 - \beta) + \beta}_t$$

(24)

The RHS of (24) is a U-shaped function of $l_t$ (or $\pi_t$), which reaches its minimum value when:

$$l_t = 1 - \beta\left(\frac{\sigma - 1}{\sigma}\right)$$

(25)

Solve for the corresponding $\pi_t$ by substituting (25) into (18):

$$\frac{\sigma(1 - \beta) + \beta}{(\sigma - 1)(1 - \beta)} = 1 + \alpha\psi(\pi_t)$$

(26)

Therefore $\lambda_t = \lambda(\pi_t)$ has the property such that for $\pi_t < \pi_t^*$, $\lambda'(\pi_t) < 0$, and for $\pi_t > \pi_t^*$, $\lambda'(\pi_t) > 0$. One can see that the connection between the interest rate policy and equilibrium now depends upon the relative values of $\pi_t$ and the steady state inflation rate $\pi^*$. In particular, we can obtain the following proposition.

**Proposition 2.** If the intertemporal elasticity of substitution is less than one, then for active interest rate rules the only equilibrium is the steady state for sufficiently low steady state inflation rate, and equilibrium is indeterminate for sufficiently high steady state inflation rate.

The intuition behind the indeterminacy result is as follows. Suppose the consumption is reduced below its steady state level, and at the same time leisure is increased above its steady state level, it follows that the nominal interest rate has to be above its steady state level. Active interest rate rules imply that the increase in the nominal interest rate is associated with an increase in the real interest rate. The rate of the marginal utility of consumption increases in response due to low intertemporal elasticity and high inflation. The inflation rate and hence the nominal interest rate will decrease. The consumption level will increase and return to its steady state level and this trajectory is consistent with an equilibrium.

It is well known that there is no consensus on the estimate of the parameter $\sigma$, and most authors believe it is greater than one. In this sense, the result in Proposition 2 may be more plausible than that in Proposition 1. In the cash-in-advance and endowment economy, one can show that Proposition 1 holds, which is similar to the results obtained in Leeper (1991) or Benhabib et al. (2001) in money in utility function models. Schmitt-Grohé and Uribe (2000) study a cash-in-advance model with endogenous labor similar to the present paper, but they assume a utility function separable in consumption and leisure ($\sigma = 1$). In all these papers, connection between the interest rate policies and the magnitude of steady state inflation rate can not be established.


Acknowledgements

I would like to thank Jess Benhabib, Mark Gertler, Yong Wang and Andrés Velasco for useful comments.

References