Returns to Scale, Market Power, and the Nature of Price Rigidity in New Keynesian Models with Self-Fulfilling Expectations

Increasing returns to scale and firms’ market power are two potential sources of sunspot expectations in neoclassical models. We show that in New Keynesian models, returns to scale and market power can have fundamentally different implications for broad macroeconomic issues, including self-fulfilling expectations, depending on the nature of price rigidity. Our findings suggest that the design of stabilization monetary policy can depend on precise knowledge about the economy’s real and nominal features. Therefore, a clear understanding of the specific economic environment and its relevance to monetary policymaking for ensuring macroeconomic stability can be an integrated part of monetary policy practice.

**JEL codes:** E12, E31, E52

**Keywords:** returns to scale, market power, price rigidity, self-fulfilling expectations.

The sources of sunspot expectations unrelated to economic fundamentals have long been studied in macroeconomics. In neoclassical models, two real features pertaining to production technology and market structure, namely, increasing returns to scale and firms’ market power, have been found as potential factors for real indeterminacy of equilibrium and sunspot-driven business cycles.\(^1\) In New Keynesian models, the focus has been on implications of nominal features

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1. For the former, see Benhabib and Farmer (1994, 1996), Farmer and Guo (1994), Wen (1998), and Benhabib and Wen (2004), among others. For the latter, see the survey by Schmitt-Grohé (1997).
pertaining to price rigidity and monetary policy for multiple equilibria and sunspot-driven fluctuations.

We study the interactions of these real and nominal features. The general thesis of this paper is that these real features can have fundamentally different implications for macroeconomic dynamics in New Keynesian models, depending on the nature of nominal price rigidity. To help demonstrate this point, we make use of two canonical sticky price models, the Rotemberg (1982) model of costly price adjustment and the Calvo (1983) model of partial price adjustment, which are commonly viewed as identical up to a linear approximation. We find that although the linearized equilibrium systems of the two models are alike in their reduced forms, as similarly demonstrated by Benhabib, Schmitt-Grohe, and Uribe (2000, 2001), among others, structural fundamentals pertaining to the underlying production technology and market structure affect the elasticity of nominal price inflation with respect to real marginal cost in fundamentally different ways across the two types of stickiness. This gives rise to a different linkage from the structural fundamentals to the slope of the reduced-form Phillips curve, depending on whether price rigidity is modeled one way or another. We analyze in some depth the differences in this connection across the two models and their causes, which can have broad implications for inflation and other aggregate dynamics.

While these differences hold in the general context of model specifications and can be relevant for a wide range of macroeconomic issues, we focus our attention in this paper on one important issue, that is, their implications for self-fulfilling expectations and sunspot-induced macroeconomic instability. To this end, we make use of these two models with a flexible forward-inflation targeting policy, under which the monetary authority sets nominal interest rate in response to variations in expected future inflation and current output. This makes it particularly convenient for one to digest our discrete-time analysis in the light of the local indeterminacy analysis of Benhabib, Schmitt-Grohe, and Uribe (2000, 2001) that is conducted in continuous time, where the monetary authority sets a nominal interest rate as a function of the instantaneous rate of inflation, since, as is now well known, the instantaneous rate of inflation in a continuous-time setting is the right derivative of the logged price level and, thus, the discrete-time counterpart of a continuous-time policy rule that sets the interest rate in response to the instantaneous rate of inflation is characterized by forward-looking policy that responds to expected future inflation. Further, as is also well known, forward-inflation targeting is prone to real indeterminacy of equilibrium, and, thus, the contrasts between the different implications of those real features of the economy for sunspot expectations under the two types of nominal rigidities are quantitatively striking with forward-looking policy. But, most importantly, forward-inflation targeting is justified on the ground of both policy effectiveness and central bank accountability

2. See, for example, Obstfeld and Rogoff (1995, p. 522, footnote 10) for an enlightening discussion of this point.
and credibility.\(^3\) On the other hand, a flexible targeting rule, which allows the nominal interest rate to respond not only to expected future inflation, but also to other endogenous variables, such as current output, is crucial for avoiding policy-induced macroeconomic instability.\(^4\) This is why such a flexible inflation-forecast targeting procedure has been advocated by many researchers\(^5\) and has become a dominant framework for monetary policymaking at central banks around the world.\(^6\) This is to say that the kind of monetary policy considered in this paper is of paramount practical interest.

We show that both the degree of returns to scale in production (including not only increasing but also constant and decreasing returns to scale) and firms’ market power can have exact opposite implications for macroeconomic stability in the Rotemberg model than in the Calvo model. Here, we summarize our results briefly. First, in the Calvo model, increasing returns to scale is a potential source of sunspot expectations,\(^7\) as in the real models, whereas decreasing returns to scale or fixed factors is potentially a stabilizing factor for the economy. In the Rotemberg model, by contrast, it is decreasing returns to scale or fixed factors that is a potential source of self-fulfilling expectations, whereas increasing returns to scale may actually be a stabilizing factor for the economy. Second, in the Calvo model, firms’ market power is a potential source of sunspot-driven fluctuations, as in the real models, provided that production technology exhibits decreasing returns to scale or fixed factors, while such market power may actually be a stabilizing factor if production technology exhibits increasing returns to scale. In the Rotemberg model, by contrast, market power of firms is a stabilizing factor for the economy regardless of the degree of returns to scale in production technology. We show these contrasts analytically for the case with labor as the only productive input and numerically for the case with endogenous capital accumulation. We show that these contrasts continue to hold in the presence of policy inertia in the form of interest rate smoothing, which is considered an

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6. This policy practice started in the industrial and middle-income countries in the late 1980s (see, e.g., Leiderman and Svensson 1995, Bernanke and Mishkin 1997, Bernanke et al. 1999), and spread to the transition and emerging-market economies in the late 1990s (see, e.g., Schaechter, Stone, and Zelmer 2000, Jonas and Mishkin 2004, Roger and Stone 2005), with more moving toward this direction. Indeed, while the U.S. Federal Reserve, the European Central Bank, and the Bank of Japan are usually viewed as having followed some implicit inflation-forecast targeting procedures, estimated forward-looking interest rate feedback rules explain well the behavior of interest rates in the U.S., Germany, and Japan (see, e.g., Chinn and Dooley 1997, Clarida and Gertler 1997, Clarida, Gali, and Gertler 1998, Orphanides 2000, Orphanides and Williams 2003, Carare and Stone 2006).

7. Xiao (2008) and Huang and Meng (2009) also use the Calvo model to analyze the implications of increasing returns to scale for macroeconomic stability under alternative interest rate policy rules, and Weder (2006) conducts a similar analysis in a monetary model with flexible prices. In a monetary model with a constant returns to scale technology and labor as the only productive input, Weder (2008) shows how the Calvo price setting may reduce the scope of indeterminate equilibria under a constant money supply rule.

These results obtained in the canonical models may convey broad implications. Understanding these implications may be important for the monetary policy practices of central banks that aim to preempt sunspot expectations and fluctuations unrelated to economic fundamentals. For one, our results reveal a clear message that the design of stabilization monetary policy can depend in a delicate way on exact knowledge about some economic fundamentals, such as those concerning the degree of returns to scale or fixed factors in production technology, market power of firms, and the nature of nominal rigidity in the economy. This poses a challenge to monetary policymakers, as existing empirical studies do not always provide unambiguous answers with regard to such real or nominal features. The fact that technological changes and institutional or policy reforms may all lead to structural changes in these specifics further complicates the issue. For instance, globalization may increase competition and lower the market power of firms, whereas protectionism policy may lead to the exact opposite; antitrust laws and regulatory policies, or lack thereof, may affect firms’ market power; technological changes may change the view on which factor input in the production process may be regarded as virtually fixed over the business cycle. On a general level, our results suggest that a clear understanding of the specific economic environment in this ever-changing world and its relevance to monetary policymaking in ensuring macroeconomic stability can be an integrated part of monetary policy design and practice at the central banks.

The remaining of the paper is organized as follows. Section 1 describes the baseline framework, with returns to scale and firms’ market power as two fundamental characteristics of the economy pertaining to production technology and market structure, and with the nature of the Rotemberg price rigidity and that of the Calvo price rigidity illustrated. To help exposition, the basic framework abstracts from capital and incorporates a baseline monetary policy rule. Section 2 derives the necessary and sufficient conditions for real determinacy of equilibrium for the Rotemberg model and for the Calvo model, and uses these conditions to establish our main results, which contrast the implications of returns to scale in production technology and market power of firms for macroeconomic stability in the Rotemberg model and in the Calvo model. This section also provides some intuition for why the two real features of the economy can have fundamentally different implications for self-fulfilling expectations across the two sticky price models. Section 3 extends the results to settings with capital and with generalized monetary policy rules. Section 4 concludes.

1. BASIC ENVIRONMENT

We start by describing the basic environment that abstracts from capital. The economy consists of a representative household and a continuum of firms that
produce differentiated intermediate goods indexed by \( j \in [0, 1] \) and that are monopolistic competitors on the markets for their products. At each date \( t \), a representative distributor combines all differentiated goods \( \{Y_{jt}\}_{j \in [0, 1]} \) into a composite goods

\[
Y_t = \left[ \int_0^1 Y_{jt}^{(e^{-1})/d} \, dj \right]^{1/(e-1)},
\]

where \( e > 1 \) is the elasticity of substitution between the differentiated goods. The distributor takes the prices \( \{P_{jt}\}_{j \in [0, 1]} \) of the individual goods as given and chooses the bundle of these intermediate goods to minimize the cost of fabricating a given quantity of the composite goods. It sells the composite goods to the household at its unit cost

\[
P_t = \left( \int_0^1 P_{1-e} \, dj \right)^{1/(1-e)},
\]

which acts also as the price level of the economy. The demand for a type \( j \) goods is given by

\[
Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-e} Y_t.
\]

This is a standard framework of monopolistic competition, in which the inverse of the elasticity of substitution between the differentiated goods also measures the market power of the monopolistically competitive firms.

At any date \( t \), the household seeks to maximize

\[
E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \frac{C_s^{1-\sigma} - 1}{1-\sigma} - \psi \frac{N_s^{1+\eta}}{1+\eta} \right), \quad \text{for } \psi > 0,
\]

where \( E_t \) denotes the expectation operator conditional on information up to date \( t \), \( \beta \in (0, 1) \) is a subjective discount factor, \( C_s \) and \( N_s \) are the household’s consumption and labor supply in period \( s \), and \( \sigma \) and \( \eta \) denote its relative risk aversion in consumption and in labor, respectively, subject to a sequence of budget constraints,

\[
C_s + \frac{E_s(D_{s,s+1}B_{s+1}) - B_s}{P_s} \leq w_s N_s + \Pi_s,
\]

for \( s \geq t \), where \( B_s \) is the household’s holding of nominal bonds at the beginning of period \( s \), \( D_{s,s+1} \) denotes the stochastic discount factor from \( s + 1 \) to \( s \), thus the gross nominal interest rate in period \( s \) is \( R_s = (E_sD_{s,s+1})^{-1} \), \( w_s \) is the real wage rate, and \( \Pi_s \) is the real profit that the household receives from the firms in period \( s \). The optimality conditions for the utility maximization problem give rise to an intratemporal consumption–labor relation

\[
w_t = \psi C_t^{\sigma} N_t^{\eta},
\]

and an intertemporal consumption Euler equation

\[
1 = \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\pi_{t+1}} \right],
\]

where \( \pi_{t+1} = P_{t+1}/P_t \) denotes the gross inflation rate from \( t \) to \( t + 1 \). We shall abstract from trend inflation so the gross steady-state inflation rate is 1.
At any date $t$, intermediate goods $j$ is produced using labor as the only input according to

$$Y_{jt} = AN_j^\theta,$$

where $A$ is the technology level, $\theta \in (0, \varepsilon / (\varepsilon - 1))$ is the degree of returns to scale in the production technology, and the upper bound on $\theta$ ensures that the firm’s profit maximization problem has a well-defined interior solution.\(^8\) Cost minimization implies that $w_t = \theta AN_t^{\theta-1} mc_t$, where $mc_t$ denotes firm $j$’s real marginal cost; thus, its real average cost is given by $ac_t = \theta N_{jt}/Y_{jt} = \theta mc_{jt}$. Defining the average real marginal cost across all firms in the economy by $mc_t = \left[\int_0^1 mc_{jt}^{1/(1-\theta)} dj\right]^{1-\theta}$, we can show that

$$w_t = \theta AN_t^{\theta-1} mc_t,$$

where $N_t = \int_0^1 N_{jt} dj$ is the aggregate labor input used by all firms. It follows that

$$\frac{mc_{jt}}{mc_t} = \left(\frac{P_{jt}}{P_t}\right)^{\frac{\varepsilon-1}{\varepsilon \theta}}. \quad (4)$$

It is worth noting that the upper bound on the degree of returns to scale in production for ensuring a well-defined interior solution for firms’ profit maximization problems is directly related to the market power of firms. To help demonstrate our general point that these real features can have fundamentally different implications for self-fulfilling expectations in New Keynesian models depending on the nature of nominal price rigidity, we employ two canonical sticky price models, the Rotemberg (1982) model of costly price adjustment and the Calvo (1983) model of partial price adjustment. In the Rotemberg model, price rigidity takes the form of a quadratic cost of nominal price adjustment faced by all firms $j \in [0, 1]$,

$$\frac{\gamma}{2} \left(\frac{P_{jt}}{P_{j,t-1}} - 1\right)^2 Y_t,$$

where $\gamma$ governs the magnitude of the cost. At any date $t$, firm $j$ seeks to maximize the expected present value of its real profit stream net of the price adjustment costs in all future periods

$$E_t \sum_{s=t}^{\infty} D_{t,s} \left[\left(\frac{P_{js}}{P_s} - ac_{js}\right)\left(\frac{P_{js}}{P_s}\right)^{-\theta} - \frac{\gamma}{2} \left(\frac{P_{js}}{P_{j,s-1}} - 1\right)^2\right] Y_s,$$

8. See, for example, Benhabib and Farmer (1994) and Huang and Meng (2009) for proof of this condition.
where $D_{t,s} = \prod_{r=t}^{s-1} D_{t+r-1,t+r}$ is an $s$-period stochastic discount factor from $t$ to $s > t$, with $D_{t,t} \equiv 1$. In a symmetric equilibrium, all firms make identical pricing decision and $mc_{jt} = mc_{t}$ for all $j \in [0, 1]$. The optimality condition for profit maximization then yields

$$\gamma (\pi_t - 1) \pi_t - \beta \gamma\left(\frac{E_t Y_{t+1}}{Y_t}\right) (E_t \pi_{t+1} - 1) E_t \pi_{t+1} + \gamma Y_t = 1 - \varepsilon + \varepsilon \theta mc_t. \tag{5}$$

In the Calvo model, price rigidity takes the form of random staggered price setting. At each date, each firm receives a random signal with a constant probability $\upsilon$ that forbids it to reset price. The signal is identically and independently distributed across firms and dates. At a date $t$, if a firm $j$ gets the chance to reset price, it chooses $P_{jt}$ for all $s \geq t$, conditional on it not getting another chance to change the price in the future and thus having to stick to the price it currently chooses, to maximize the expected present value of its real profit stream

$$E_t \sum_{s=t}^{\infty} \upsilon^{s-t} D_{t,s} \left(\frac{P_{js}}{P_s} - ac_{js}\right) \left(\frac{P_{js}}{P_s}\right)^{-\varepsilon} Y_s,$$

giving rise to the following optimal pricing decision:

$$P_{jt} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{s=t}^{\infty} \upsilon^{s-t} D_{t,s} P_{1+s}^1 Y_s \theta mc_{js}}{E_t \sum_{s=t}^{\infty} \upsilon^{s-t} D_{t,s} P_{1+s}^1 Y_s}. \tag{6}$$

With the large number of firms, at each date there is a fraction $\upsilon$ of randomly selected firms that cannot reset prices and a fraction $(1 - \upsilon)$ that can. The price level at $t$ is then

$$P_t = \left[\upsilon P_{t-1}^{1-\varepsilon} + (1 - \upsilon) P_{jt}^{1-\varepsilon}\right]^\frac{1}{1-\varepsilon}. \tag{7}$$

To close the models, motivated by the theoretical justifications and policy practices at central banks around the world, as discussed in the Introduction section, especially, footnotes 2–5, we consider a flexible forward-inflation targeting rule under which the monetary authority sets nominal interest rate in response to the deviation of expected future inflation from the inflation target (equal to the steady-state inflation rate) and variation in current output,

$$R_t = R \left(E_t \pi_{t+1}\right)^\tau \left(\frac{Y_t}{Y}\right)^{\omega}, \tag{8}$$

where $R$ and $Y$ denote, respectively, the steady-state nominal interest rate and real output. We allow the possibility for the interest rate to respond also to the deviation of current output from the steady state to capture the idea of flexible inflation targeting,
as the variation in current output may bring with it a pressure or relief on future inflation.\textsuperscript{9}

2. MAIN RESULTS AND SOME INTUITION

It can be verified that there is a unique steady state regardless of the nature of price rigidity. For local determinacy analysis, we can thus examine a log-linearized system of equilibrium conditions around the steady state. Throughout the rest of the paper, we shall use a variable with a hat to denote the percentage deviation of that variable in level from its steady-state value.

We begin by deriving the log-linear version of (2)

$$\hat{R}_t - E_t \hat{\pi}_{t+1} = \sigma (E_t \hat{\pi}_{t+1} - \hat{\pi}_t),$$

(9)

and by combining the log-linear versions of (1) and (3) to get

$$\hat{m}_t = \left( \frac{\eta + 1}{\theta} + \sigma - 1 \right) \hat{y}_t,$$

(10)

where we have used the log-linear version of the market clearing condition for the composite goods, $\hat{c}_t = \hat{y}_t$, in rewriting (9) and (10). The log-linearized version of (8) is simply

$$\hat{R}_t = \tau E_t \hat{\pi}_{t+1} + \omega \hat{y}_t.$$  

(11)

These log-linear equilibrium conditions hold for both the Calvo model and the Rotemberg model.

The log-linearized version of (5) gives rise to the New Keynesian Phillips curve for the Rotemberg model, and the log-linearized versions of (6), (7), and (4) give rise to the New Keynesian Phillips curve for the Calvo model, which share the same functional form,

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda_i \hat{m}_t,$$

(12)

but what differentiates the two types of price rigidities lies with the Phillips curve slope that is linked to the deep parameters governing returns to scale in production

\textsuperscript{9} We have also examined the case in which the interest rate is allowed to respond to expected future output changes and the findings once again support our general view that the real features of the economy can have fundamentally different implications for self-fulfilling expectations in New Keynesian models depending on the nature of nominal price rigidity. We do not present these results here in order to conserve space.
technology and monopolistic power of firms due to market structure in fundamentally different ways in the Rotemberg model than in the Calvo model, or, specifically,

\[ \lambda_R = \frac{\varepsilon - 1}{\gamma}, \]

for the Rotemberg model, and

\[ \lambda_C = \frac{(1 - \upsilon)(1 - \upsilon \beta)}{\upsilon} \frac{1}{1 + \varepsilon \frac{1-\delta}{\delta}}, \]

for the Calvo model.

Equations (9)–(12) constitute a complete set of equilibrium conditions that can be used for local determinacy analysis. With some algebra, we can establish the necessary and sufficient conditions for real determinacy for the Rotemberg model and for the Calvo model as follows:

\[ - (1 - \beta) \omega < B \lambda_i (\tau - 1) < (1 + \beta)(2 \sigma + \omega), \]

where

\[ B \equiv \eta + \frac{1}{\theta} + \sigma - 1 \]

measures the elasticity of real marginal cost with respect to real output in light of (10). We shall consider the regular situation in which this elasticity is positive, so expansion in real output will put an upward pressure on real marginal cost, and vice versa. Given \( \varepsilon > 1 \) and \( \theta \in (0, \varepsilon/(\varepsilon - 1)) \), a sufficient condition for this to always be the case is for the inverse of the monopolistic markup \((\varepsilon - 1)/\varepsilon\) to be bounded below by \((1 - \sigma)/(1 + \eta)\), which is met for almost all empirically plausible values of the parameters. The elasticity of current inflation with respect to current real marginal cost, that is, the slope of the New Keynesian Phillips curve, is also positive in both the Rotemberg model and the Calvo model, that is, both \( \lambda_R > 0 \) and \( \lambda_C > 0 \).

Now we are ready to show that both the degree of returns to scale in production and firms’ market power can have exact opposite implications for macroeconomic stability in the Rotemberg model than in the Calvo model. Examining the necessary and sufficient conditions for determinacy (13) and how \( B, \lambda_R, \) and \( \lambda_C \) depend on \( \theta \) and \( \varepsilon \) reveals the following contrasts. First, in the Calvo model, increasing returns to scale is a potential source of real indeterminacy of equilibrium, while decreasing returns or fixed factors may be a stabilizing factor for the economy. In the Rotemberg model, decreasing returns or fixed factors is a potential source of sunspot expectations, while increasing returns may help stabilize the economy. Second, in the Calvo model, market power of firms, as is measured by the monopolistic markup \( \varepsilon/(\varepsilon - 1) \) and,

10. Recall that the upper bound on \( \theta \) is required in order to satisfy the second-order condition for firms’ profit maximization problem.
thus, is inversely related to $\varepsilon$, is a potential source of self-fulfilling expectations if production technology exhibits decreasing returns or fixed factors, while such market power may help stabilize the economy if production technology exhibits increasing returns to scale. In the Rotemberg model, market power of firms is a stabilizing factor for the economy regardless of the degree of returns to scale in production technology. These results sharply contrast the fundamentally different implications of returns to scale and market power for macroeconomic stability in the Rotemberg model than in the Calvo model.

To gain some insights into these contrasts, we start by observing that, in the Rotemberg model, the timings of price setting are synchronized across firms, and thus, in a symmetric equilibrium, all firms set the same price, so that there are no relative price fluctuations. As a result, here there are no concerns about profit losses resulting from fluctuations in relative prices. In the Calvo model, instead, the timings of price setting are staggered across firms, and, thus, relative price fluctuations arise in equilibrium, and these relative price fluctuations affect firms’ profits. As a result, curvature of profits with respect to relative prices plays a role in firms’ pricing behaviors here. The presence of relative price fluctuations with an allocative role in the Calvo model, but lack thereof in the Rotemberg model, gives rise to differences between these two models in their implications for how returns to scale and market power affect the response of inflation to changes in marginal cost, and so the slope of the Phillips curve. This has important consequences for endogenous inflation inertia, which is key for macroeconomic dynamics in the New Keynesian models.

It is worth noting that these contrasts stand in the general context of model specifications, without appealing to monetary policy or household preferences, and they can be relevant for a wide range of macroeconomic issues. In particular, they may help us understand why the two real features of the economy can have different implications for self-fulfilling expectations across the two sticky price models that are workhorses in the New Keynesian literature. Here, it is useful to note that greater inflation inertia can follow from a smaller elasticity of inflation with respect to marginal cost, or a smaller elasticity of marginal cost with respect to output, as each of which implies a smaller change in inflation in response to a given change in aggregate demand condition. Under our specification of a forward-looking monetary policy rule, it is also useful to recall that the insights provided in Kurozumi and Zandweghe (2008) and Huang, Meng, and Xue (2009) suggest that greater endogenous inflation inertia can make the forward-inflation targeting policy more effective in avoiding sunspot-driven fluctuations.

To put this into perspective, we now discuss the effect of returns to scale on endogenous inflation inertia. We note that the parameter $\theta$ affects inflation inertia through two channels: its effect on the elasticity of marginal cost with respect to output, $B$, as shown by equation (10), and its effect on the elasticity of inflation with respect to marginal cost, or, the slope of the New Keynesian Phillips curve, $\lambda_i$, for $i = R, C$, as shown by equation (12). The first effect is the same across the two models, and a large $\theta$ tends to stabilize the economy through this channel by inducing a smaller elasticity of marginal cost to output, which tends to give rise to
greater inflation inertia. The second effect is muted in the Rotemberg model, as $\lambda_R$ is independent of $\theta$. Thus, in the Rotemberg model, real indeterminacy of equilibrium is less likely to occur, the greater is $\theta$. This explains why increasing returns may help stabilize the economy while decreasing returns or fixed factors is a potential source of sunspot expectations, if price rigidity is of the Rotemberg type. In the Calvo model, by contrast, the second effect is active and works against the first effect, as $\lambda_C$ is an increasing function of $\theta$. That is, in the Calvo model, a large $\theta$ tends to destabilize the economy through the second channel by inducing a greater elasticity of inflation to marginal cost, which tends to give rise to smaller inflation inertia. This second effect dominates the first one, and this is why increasing returns is a potential source of self-fulfilling expectations while decreasing returns or fixed factors can be a stabilizing factor for the economy, if price rigidity is of the Calvo type.

We turn next to the effect of market power of firms on endogenous inflation inertia. Since $B$ is independent of $\varepsilon$, market power of firms affects inflation inertia solely through its effect on the elasticity of inflation with respect to marginal cost, $\lambda_i$, for $i = R, C$, as shown by equation (12). We note that $\lambda_R$ is an increasing function of $\varepsilon$. Thus, in the Rotemberg model, a greater market power of firms (corresponding to a smaller value of $\varepsilon$) acts to stabilize the economy by inducing a smaller elasticity of inflation with respect to marginal cost to generate greater inflation inertia. This explains why market power of firms can help preempt sunspot expectations if price rigidity is of the Rotemberg type. We note now that $\lambda_C$ is increasing in $\varepsilon$ if $\theta > 1$, but is decreasing in $\varepsilon$ if $\theta < 1$. Thus, in the Calvo model, a greater market power of firms acts to stabilize the economy by inducing a smaller elasticity of inflation with respect to marginal cost to generate greater inflation inertia if production technology exhibits increasing returns, but it acts in the opposite way to destabilize the economy by inducing a greater elasticity of inflation with respect to marginal cost to generate smaller inflation inertia if production technology exhibits decreasing returns or fixed factors. This explains why market power of firms may help preempt self-fulfilling expectations under increasing returns to scale production technology, but may in itself become a source of sunspot expectations with decreasing returns to scale or fixed factors, if price rigidity is of the Calvo type.

3. WITH ENDOGENOUS CAPITAL ACCUMULATION

We now extend the baseline setting to include capital as another variable input, along with labor. With this extension, closed-form analytical results are hard to come by, but the numerical exercises below confirm our conclusions based on the analytical results obtained in the previous section for the case without capital.

To be specific about the extended setting, on the household side, any quantity of the composite goods purchased by the household can now be not only consumed but invested to accumulate capital stock that the household can rent to firms in a
competitive capital market. The household’s budget constraint in period $t$ is modified as

$$C_t + I_t + \frac{E_t(D_{t+1} - B_t)}{P_t} \leq w_t N_t + r_t K_t + \Pi_t,$$

where $I_t$ and $K_t$ are the household’s investment and capital supply, respectively, and $r_t$ is real capital rental rate. The law of motion for capital is

$$K_{t+1} = I_t + (1 - \delta) K_t,$$

where $\delta$ is the capital depreciation rate. The household’s utility maximization conditions now also include a capital Euler equation

$$1 = E_t \left\{ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} (r_{t+1} + 1 - \delta) \right\}.$$  \hspace{1cm} (14)

On the firm side, we now have the following production function for intermediate goods $j$:

$$Y_{jt} = A \left( K_{jt}^{\alpha} N_{jt}^{1-\alpha} \right)^{\theta}, \quad \text{for } \alpha \in (0, 1),$$

where $\theta \in (0, \varepsilon/(\varepsilon - 1))$ still measures the degree of returns to scale at the individual firm level. Cost minimization by firm $j$ involves choosing $N_{jt}$ and $K_{jt}$ to minimize $w_t N_{jt} + r_t K_{jt}$ subject to the above production function, taking the real wage rate $w_t$ and real capital rental rate $r_t$ as given. The resulting factor demand conditions imply the following relations between firm $j$’s real marginal cost, $mc_{jt}$, and its output, $Y_{jt}$, its factor inputs, $N_{jt}$ and $K_{jt}$, and factor prices, $w_t$ and $r_t$,

$$mc_{jt} = \frac{1}{1 - \alpha} \frac{w_t}{\theta Y_{jt}/N_{jt}} = \frac{1}{\alpha \theta} \frac{r_t}{Y_{jt}/K_{jt}}.$$  \hspace{1cm} (15)

Using these relations for the individual firms, we can also derive the following relations between the average real marginal cost across all firms, $mc_i$, and total output, $Y_t$, total factor inputs used by all firms, $N_i = \int_0^1 N_{jt} dj$ and $K_i = \int_0^1 K_{jt} dj$, and factor prices, $w_t$ and $r_t$:

$$mc_i = \frac{1}{1 - \alpha} \frac{w_t}{\theta Y_t/N_i} = \frac{1}{\alpha \theta} \frac{r_t}{Y_t/K_i}.$$  \hspace{1cm} (15)

It follows that

$$\frac{mc_{jt}}{mc_i} = \left( \frac{P_{jt}}{P_i} \right)^{\frac{\alpha \theta}{1 - \alpha}} = \left( \frac{Y_{jt}}{Y_t} \right)^{\frac{\alpha \theta}{1 - \alpha}}.$$  \hspace{1cm} (16)
The profit maximization problems and the resulting optimal pricing equations in the two models remain the same as in the baseline setting.

The system of log-linearized equilibrium conditions that can be used for local determinacy analysis consists of the policy rule (11) and the New Keynesian Phillips curve (12), which all remain the same as in the baseline setting, plus a modified consumption Euler equation, a capital Euler equation, and a modified market clearing condition for the composite goods,

\[
\hat{R}_t - E_t \hat{\pi}_{t+1} = \sigma (E_t \hat{c}_{t+1} - \hat{c}_t),
\]

\[(\tilde{\delta} - 1) \sigma E_t \hat{c}_{t+1} - \delta \hat{k}_{t+1} + \tilde{\delta} (1 + \eta) E_t \hat{n}_{t+1} = -\sigma \hat{c}_t,
\]

\[
\hat{k}_{t+1} = -c_k \hat{c}_t + [\alpha \theta (\delta + c_k) + 1 - \hat{\delta}] \hat{k}_t + (1 - \alpha) \theta (\delta + c_k) \hat{n}_t,
\]

where \(\tilde{\delta} \equiv 1 - \beta (1 - \delta)\) and \(c_k \equiv (\varepsilon \beta) / [\alpha \theta \beta (\varepsilon - 1)] - \delta\).

In this extended setting with endogenous capital accumulation, the contrasts between the implications of returns to scale and firms’ market power for real indeterminacy of equilibrium across the Rotemberg model and the Calvo model presented in the baseline setting continue to hold. To get a quantitative feel about the contrasts, we now assign values to the fundamental parameters in the economy.

We set \(\alpha\) to 0.33 so that the share of payment to capital in value-added productive factors is equal to one-third, as in the National Income and Product Account. Given that one period in our model corresponds to one quarter of a year, we set \(\beta = 0.99\) to be consistent with a steady-state annualized real interest rate of 4%, and we set \(\delta\) to 0.02 to match the steady-state annual capital depreciation rate of 8%. These are standard parameter values used in the literature. While some studies in the literature suggest that relative risk aversion in consumption, \(\sigma\), can be as low as 0 or as high as 30, the general consensus is that it lies between 1 and 10. Our results are robust to the choice of \(\sigma\) in its empirically reasonable range, and thus we fix \(\sigma\) at 2.

The inverse of \(\eta\), the relative risk aversion in labor, corresponds to the Frisch elasticity of labor supply, which has been estimated in many empirical studies. The evidence obtained based on different sources of data, frequencies of time, sample periods, aggregation levels, substitution margins (intensive versus extensive), seasonality adjustments, and estimation procedures suggests that the elasticity lies between 0.05 and 2.\(^{11}\) This suggests a value for \(\eta\) between 0.5 and 20. On the other side, a value of \(\eta = 0\), corresponding to an infinite labor supply elasticity, is often assumed in many

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11. Many of these studies based on life-cycle data of hours worked (intensive margin) by men at annual or lower frequencies find an elasticity in the range of 0.05 to 0.3, as summarized by Pencavel (1986). Evidence on a low elasticity is also found by Altonji (1986), Ball (1990), and Card (1994). Using monthly data from the Denver Income Maintenance Experiment, MaCurdy (1985) finds that the (intensive) elasticity can be above 0.3 though it is below 0.7. Rupert, Rogerson, and Wright (2000) obtain similar estimates taking home production into account. Using triannual micropanel data of the Survey of Income and Program Participation, Kimmel and Kniesner (1998) estimate an hours worked elasticity (intensive margin) of 0.5 and an employment elasticity (extensive margin) of 1.5. Mulligan (1998) shows that the
studies on indeterminacy (see, e.g., Benhabib and Eusepi 2005, Carlstrom and Fuerst 2005, Kurozumi and Zandweghe 2008). Our results are robust to the choice of \( \eta \) in its empirically reasonable range, and thus we fix \( \eta \) at 1, corresponding to a unit labor supply elasticity, as is standard in the macroeconomic literature.

A reasonable range for \( \varepsilon \), the elasticity of substitution between differentiated goods, is from 4 to 20, in light of the empirical studies by Domowitz, Hubbard, and Petersen (1986), Shapiro (1987), Basu (1996), Rotemberg (1996), Rotemberg and Woodford (1997), Basu and Kimball (1997), Linnemann (1999), and Basu and Fernald (1994, 1995, 1997, 2000, 2002). We set our benchmark value of \( \varepsilon \) to 10, as is common in the literature, while we examine how our indeterminacy results in the two sticky price models are affected by varying \( \varepsilon \) in its empirically relevant range. We also recall here that the profit maximization problem has a well-defined interior solution as long as the degree of returns to scale \( \theta \) is bounded above by the steady-state monopolistic markup \( \varepsilon / (\varepsilon - 1) \). The benchmark value that we choose for \( \theta \) is 1, corresponding to the case with constant returns to scale in production technology, which meets the upper bound regardless of the value considered for \( \varepsilon \). For the benchmark value of \( \varepsilon \) we can consider a degree of increasing returns to scale as high as 1.05 when examining the case with \( \varepsilon = 20 \), in order to guarantee a well-defined interior solution. We consider a possible extent of decreasing returns to scale, or fixed factors in production technology, equal to 0.33, similarly as in Chari, Kehoe, and McGrattan (2000). We shall examine how our indeterminacy results in the two sticky price models are affected by varying \( \theta \) in a relevant range.

We set \( \nu \), the hazard rate of price adjustment in the Calvo model, to 0.57, which lies in the middle of its empirical estimates (see, e.g., Galí and Gertler 1999, Galí, Gertler, and Lopez-Salido 2001, 2003, Sbordone 2002, Bils, Klenow, and Kryvtsov 2003, Bils and Klenow 2004, Alvarez et al. 2005, Christiano, Eichenbaum, and Evans 2005, Dhyne et al. 2005, Levin et al. 2005, Klenow and Krystov 2008, Nakamura and Steisson 2008), and which is close to the values used in studies on indeterminacy in the Calvo setting (see, e.g., Carlstrom and Fuerst 2005, Sveen and Weinke 2005, 2007, Kurozumi and Zandweghe 2008). We set \( \gamma \), the quadratic price adjustment cost parameter in the Rotemberg model, to 30.424, so that the elasticity of price to marginal cost in the Rotemberg model and in the Calvo model is the same under the benchmark values of the other parameters in the economy, and as is close to the values used in studies on indeterminacy in the Rotemberg setting.

With these values of the fundamental parameters in hand, we can start to search for the values of \( \tau_{\pi} \) and \( \tau_{y} \) that ensure a unique equilibrium. We search in the range \([0, 5]\) for \( \tau_{\pi} \) and \([0, 1]\) for \( \tau_{y} \), which cover most of the empirical estimates of the two elasticity can lie in the 1–2 range. Many other studies, using multiindustry panel or macro-level data (as reported by Browning, Hansen, and Heckman 1999), using industry-specific data (see, e.g., Treble 2003, Carrington 1996, Oettinger 1999), and using data at several different frequencies (see, e.g., Abowd and Card 1989) or taking into account seasonal variation in total hours worked (see, e.g., Barsky and Miron 1989, Hall 1999), also provide corroborating evidence that helps reach a consensus that the elasticity lies between 0.05 and 2.
Fig. 1. Comparison of Determinacy Regions for the Rotemberg Model and the Calvo Model with Different Degrees of Returns to Scale in Production.

Notes: Upper bound (solid line) and lower bound (broken line) for policy’s response to future inflation \(\tau\) (vertical axis) as a function of policy’s response to current output \(\omega\) (horizontal axis)—equilibrium is indeterminant in the areas outside the determinacy regions.

Our numerical exercises under endogenous capital accumulation confirm our analytical results obtained for the case that abstracts from capital. Results from these numerical exercises are reported in Figures 1–6. As before (see equation (13)), the determinacy region is characterized by a lower bound and an upper bound on policy’s...
response to future inflation $\tau$ (the vertical axis) as a function of policy’s response to current output $\omega$ (the horizontal axis), while equilibrium is indeterminant in the area outside the determinacy region.

Figure 1 displays the determinacy (indeterminacy) regions for the Rotemberg model and the Calvo model with different degrees of returns to scale in production technology. As the figure illustrates, as we increase the degree of returns to scale from 0.67 to 1, and then to 1.1, while keeping the other parameters at their benchmark.
values, the determinacy region gets enlarged and the indeterminacy region shrinks if price rigidity is of the Rotemberg type (the left panel), while the opposite is true if price rigidity is of the Calvo type (the right panel). This confirms our original finding that increasing returns to scale is a potential source of sunspot expectations and decreasing returns to scale or fixed factors is potentially a stabilizing factor in
the Calvo model, whereas it is decreasing returns to scale or fixed factors that is a potential source of self-fulfilling expectations and increasing returns to scale may actually be a stabilizing factor in the Rotemberg model.

Figure 2 plots the determinacy (indeterminacy) regions for the Rotemberg model and the Calvo model with varying extents of the market power of firms under different
degrees of returns to scale in production technology. As the figure demonstrates, as we increase the elasticity of substitution between differentiated goods from 4 to 10, and then to 20, that is, as we decrease the steady-state monopolistic markup of price over marginal cost from 33% to 11%, and then to 5.26%, the determinacy region shrinks and the indeterminacy region gets enlarged in the Rotemberg model, regardless of whether production technology exhibits decreasing, constant, or increasing returns.
Fig. 6. Determinacy Region in the Calvo Model with Varying Extents of Firms’ Market Power under Different Degrees of Returns to Scale in Production and Policy Inertia.

NOTES: Upper bound (solid line) and lower bound (broken line) for policy’s response to future inflation $\tau$ (vertical axis) as a function of policy’s response to current output $\omega$ (horizontal axis)—equilibrium is indeterminant in the area outside the determinacy region.

to scale (the left panel), while the same is true in the Calvo model if production technology exhibits increasing returns to scale but the opposite holds if production technology exhibits decreasing returns to scale (the right panel). The other parameters are maintained at their benchmark values. This confirms our original finding that firms’ market power is a stabilizing factor in the Rotemberg model regardless of the degree of returns to scale in production technology, while, in the Calvo model,
firms’ market power is also a stabilizing factor under increasing returns to scale in production but it becomes a potential source of sunspot-driven fluctuations if production technology exhibits decreasing returns to scale or fixed factors.

As a further robustness check, we also examine how policy inertia affects the above contrasts. To model policy inertia, we replace the baseline policy rule (8) by the following interest rate rule

\[ R_t = \phi R_{t-1} R^{1-\phi} (E_t \pi_{t+1})^\tau \frac{Y_t}{Y} + \omega \tilde{\pi}_t, \]

where \( \phi \) measures the degree of interest rate smoothing in the policy rule. With some algebra, which we skip here in order to conserve space, we show that the local determinacy analysis can be done by examining the following system of five first-order linear difference equations in three jump variables, \( \hat{c}_t, \hat{n}_t, \) and \( \hat{\pi}_t, \) and two predetermined variables, \( \hat{k}_t \) and \( \hat{R}_{t-1}, \)

\[
\begin{pmatrix}
(\tilde{\delta} - 1) & \sigma & \tilde{\delta} (1 + \eta) & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta & 0 \\
\sigma & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & -\tau & 1 \\
\end{pmatrix}
\begin{pmatrix}
\hat{c}_{t+1} \\
\hat{k}_{t+1} \\
\hat{n}_{t+1} \\
\hat{\pi}_{t+1} \\
\hat{R}_{t}
\end{pmatrix}
\begin{pmatrix}
E_t \hat{c}_t + 1 \\
E_t \hat{n}_t + 1 \\
E_t \hat{\pi}_t + 1 \\
\hat{R}_t - 1
\end{pmatrix}
\]

for \( i = R, C, \) corresponding to the Rotemberg model and the Calvo model, respectively, where

\[ c_k = \frac{\varepsilon}{\varepsilon - 1} \frac{\tilde{\delta}}{\alpha \theta \beta} - \delta. \]

Figures 3–6 display the determinacy (indeterminacy) regions for the Rotemberg model and the Calvo model with various degrees of interest rate smoothing in the policy rule, firms’ market power, and returns to scale in production technology. As can be seen from the figures, the determinacy region in the face of policy inertia is once again characterized by a lower bound and an upper bound on policy’s response to future inflation \( \tau \) (the vertical axis) as a function of policy’s response to current output \( \omega \) (the horizontal axis), while equilibrium is indeterminant in the area outside the determinacy region. As is clear, the contrasts between these two models presented above in terms of the implications of the two real features of the economy for sunspot expectations and macroeconomic instability continue to hold in the face of policy inertia.

The above results obtained in the canonical models may convey broad implications. Sorting out these implications may be important for the monetary policymakers that
aim to preempt sunspot expectations and fluctuations unrelated to economic funda-
mentals. For one, our results reveal a clear message that the design of stabilization
monetary policy can depend in a delicate way on exact knowledge about some eco-
nomic fundamentals, such as those concerning the degree of returns to scale or fixed
factors in production technology, market power of firms, and the nature of nominal
rigidity in the actual economy.

4. CONCLUSION

Increasing returns to scale in production and the market power of firms are two real
features identified in the literature as the potential sources of sunspot expectations in
neoclassical models. We have shown in this paper that both the degree of returns to
scale (including not only increasing but also constant and decreasing returns to scale)
and firms’ market power can have fundamentally different implications for broad
macroeconomic issues, in particular, self-fulfilling expectations, in New Keynesian
models, depending on the nature of nominal price rigidity.

As we have discussed earlier, the presence of relative price fluctuations with an
allocative role in the Calvo model due to unsynchronized timings of price setting, but
lack thereof in the Rotemberg model due to synchronized timings of price setting, is
what lies behind the contrasts between these two canonical New Keynesian models
in their implications for how the real features of the economy influence the elasticity
of nominal price inflation with respect to real marginal cost, and thus the slope of the
Phillips curve. This, as we have shown in the paper, has important consequences for
endogenous inflation inertia, which plays a crucial role in general macroeconomic
issues, including self-fulfilling expectations and sunspot-driven fluctuations. As we
have emphasized before, these contrasts hold in the general context of model spec-
ifications, without appealing to monetary policy or household preferences, and, as
such, they can be relevant for a broad range of macroeconomic issues, other than
self-fulfilling expectations.

Here we also want to note that, the contrasts between the two models may go
beyond returns to scale and market power, to other real features of the economy,
such as firm-specific factors. For instance, Sveen and Weinke (2005, 2007) make an
important observation that a Calvo model with firm-specific capital is isomorphic to
a Calvo model without firm-specific capital but stickier prices. The insights that we
have developed in this paper in the contrasts between Calvo pricing and Rotemberg
pricing suggest that this is not true with the Rotemberg type of price stickiness. The
reason is simple: with synchronized timings of price setting, firm-specific capital
plays no role in the Rotemberg model, since firms will choose the same level of
capital, just like they will set the same level of price, in a symmetric equilibrium. We
have considered above the case in which capital is specific but variable at the firm
level, as in Sveen and Weinke (2005, 2007). If instead, capital is firm specific and
constant, this is similar to the case of decreasing returns to scale (or fixed factors).
Then, as we have shown in this paper, this feature has different implications in the Calvo model than in the Rotemberg model.

One message that one can take out of the results presented in this paper is that the design of stabilization monetary policy can depend in a delicate way on exact knowledge about some economic fundamentals, such as that concerning the degree of returns to scale or fixed factors in production technology, the market power of firms, and the nature of nominal rigidity. This poses a challenge to monetary policymakers, as existing empirical studies do not always provide unambiguous answers with regard to such real or nominal features of the actual economy. The fact that technological changes, institutional or policy reforms may all result in structural changes in these specifics further complicates the issue. If policymakers misperceive these fundamental features of the economy, then policy intended to preempt sunspot expectations may lead instead to economic fluctuations unrelated to fundamentals. To meet this challenge, a clear understanding of the specific economic environment in this ever-changing world and its relevance to monetary policymaking in ensuring macroeconomic stability can become an integrated part of monetary policy design and practice at central banks.

LITERATURE CITED


