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# Increasing returns and unsynchronized wage adjustment in sunspot models of the business cycle ☆

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**Abstract**

A challenge to models of equilibrium indeterminacy based on increasing returns is that required increasing returns for generating indeterminacy can be implausibly large and rise quickly with the relative risk aversion in labor. We show that unsynchronized wage adjustment via a relative wage effect can both lower the required degree of increasing returns for indeterminacy to a plausible level and make it invariant to the relative risk aversion in labor. Consequently, indeterminacy and sunspot-driven fluctuations can emerge for plausible increasing returns regardless of the relative risk aversion in labor. Our model generates reasonable dynamics in terms of matching the business cycle, and sunspot shocks become more important with labor market friction.



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*Keywords:* Increasing returns; Relative wage; Risk aversion in labor; Indeterminacy; Sunspot

## 1. Introduction

It has long been argued that economies of scale can be a potential source of sunspot-driven business cycle fluctuations (e.g., Benhabib and Farmer [13]). In one-sector models with variable

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capacity utilization mild increasing returns to scale in production may generate an indeterminate equilibrium and sunspot-driven business cycles under the assumption of zero relative risk aversion in labor (e.g., Wen [59]; Benhabib and Wen [15]); but, even with a tiny relative risk aversion in labor the degree of increasing returns required for indeterminacy can be implausibly large, and it increases sharply with the relative risk aversion in labor. A similar problem also exists for two-sector models (e.g., Benhabib and Farmer [14]). This poses a challenge to the plausibility of sunspot-driven business cycle fluctuations based on increasing returns to scale in production.

One point of this paper is that unsynchronized wage adjustment via a relative wage effect can both lower the required degree of increasing returns for indeterminacy to an empirically plausible level and make it invariant to the relative risk aversion in labor. As a consequence, an indeterminate equilibrium and thus sunspot-driven business cycle fluctuations may emerge for empirically plausible increasing returns regardless of the value of the relative risk aversion in labor. We show that the impulse responses of the model to demand shocks under indeterminacy are reasonable in terms of matching the business cycle and sunspot shocks become more important due to the presence of labor market frictions.

The rest of the paper is organized as follows. Section 2 sets up the model and defines an equilibrium. This section also derives the equilibrium conditions along with a log-linearized equilibrium system for local determinacy analysis. Section 3 describes model calibration and reports our indeterminacy results. Section 4 provides some intuitions for the results. Section 5 presents the model's general solution and impulse responses to fundamental shocks under determinacy and to both fundamental shocks and sunspot shocks under indeterminacy. Section 6 contains a brief concluding remark. Appendices A and B describe some alternative model structures that deliver identical results.

## 2. A one-sector model of monopolistic competition

We augment the Benhabib and Farmer's [13] model of monopolistic competition in the goods market with variable capacity utilization in production, in the spirit of Greenwood, Hercowitz, and Huffman [27], Wen [59], and Benhabib and Wen [15], and money in the utility function as an asset to provide liquidity services. We model monopolistic competition in the labor market also, which allows us to study the effect of unsynchronized wage adjustment in a standard way.

The economy is populated with a continuum of households, each endowed with a differentiated labor skill indexed by $i \in [0, 1]$, whereas it consists of a continuum of firms, each producing a differentiated intermediate good indexed by $j \in [0, 1]$.

At each date $t$, perfectly competitive retailers combine the differentiated intermediate goods $\{y_t(j)\}_{j\in[0,1]}$ into a composite final good $y_t$, using a Dixit–Stiglitz [24] technology,

$$y_t = \left[ \int_0^1 y_t(j)^{\frac{1}{\varepsilon}}\, dj \right]^{\varepsilon}, \tag{1}$$

where $\varepsilon > 1$, with $\varepsilon/(\varepsilon - 1)$ measuring the elasticity of substitution between the individually differentiated goods. The retailers take the prices $\{P_t(j)\}_{j\in[0,1]}$ for the intermediate goods as given and choose the bundle of the intermediate goods $\{y_t(j)\}_{j\in[0,1]}$ to minimize the cost $\int_0^1 P_t(j) y_t(j)\, dj$ of fabricating a given quantity of the final good $y_t$, subject to (1). The resultant demand schedule for a type $j$ intermediate good is given by

$$y_t^d(j) = \left[\frac{P_t(j)}{P_t}\right]^{\frac{\varepsilon}{1-\varepsilon}} y_t, \tag{2}$$

where $P_t$ is the resultant cost of fabricating one unit of the composite final good given by

$$P_t = \left[\int_0^1 P_t(j)^{\frac{1}{1-\varepsilon}}\, dj\right]^{1-\varepsilon}, \tag{3}$$

which is the price at which the retailers sell the composite final good to the households, and which also corresponds to the general price level of the economy.

There are also perfectly competitive distributors that combine the differentiated labor services $\{n_t(i)\}_{i\in[0,1]}$ into a composite labor service $n_t$, using a Dixit–Stiglitz [24] technology,

$$n_t = \left[\int_0^1 n_t(i)^{\frac{\epsilon-1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

where $\epsilon > 1$ measures the elasticity of substitution between the individually differentiated labor services. The distributors take the nominal wage rates $\{W_t(i)\}_{i\in[0,1]}$ for the differentiated labor services as given and choose the bundle of the differentiated labor services $\{n_t(i)\}_{i\in[0,1]}$ to minimize the cost $\int_0^1 W_t(i)n_t(i)\, di$ of fabricating a given quantity of the composite labor service $n_t$, subject to (4). The resultant demand schedule for a type $i$ labor service is given by

$$n_t^d(i) = \left[\frac{W_t(i)}{W_t}\right]^{-\epsilon} n_t, \tag{5}$$

where $W_t$ is the resultant cost of fabricating one unit of the composite labor service given by

$$W_t = \left[\int_0^1 W_t(i)^{1-\epsilon}\, di\right]^{\frac{1}{1-\epsilon}}, \tag{6}$$

which is the nominal wage rate at which the distributors charge the firms for hiring the composite labor service, and which also corresponds to the general wage level of the economy.

Units of the composite final good $y_t(i)$ purchased by a household $i$ from the retailers can either be consumed, or be invested to accumulate capital stock,

$$y_t(i) = c_t(i) + \bigl\{k_{t+1}(i) - \bigl[1 - \delta\bigl(u_t(i)\bigr)\bigr]k_t(i)\bigr\}, \tag{7}$$

where $c_t(i)$ is consumption, $k_t(i)$ is capital, $u_t(i) \in (0,1)$ is capacity utilization rate, and the capital depreciation rate is an increasing and convex function of the capacity utilization rate,

$$\delta\bigl(u_t(i)\bigr) = \frac{u_t(i)^\phi}{\phi}, \quad \text{for } \phi > 1, \tag{8}$$

as in Greenwood, Hercowitz, and Huffman [27], Wen [59], and Benhabib and Wen [15]. The budget constraint facing the household in period $t$ is

$$y_t(i) + \frac{M_t(i)}{P_t} = \frac{W_t(i)}{P_t} n_t(i) + r_t u_t(i) k_t(i) + \frac{M_{t-1}(i)}{P_t} + f_t(i) + \tau_t(i), \tag{9}$$

where $M_t(i)$ is nominal money balances, $r_t$ is real capital rental rate, and $f_t(i)$ and $\tau_t(i)$ are real profit and real transfer from the firms and the government, respectively. The household's goal at date $t$ is to maximize

$$\mathrm{E_t}\sum_{s=t}^{\infty}\rho^{s-t}\left[\log c_s(i)+\psi_1\log\frac{M_s(i)}{P_s}-\psi_2\frac{n_s(i)^{1+\chi}}{1+\chi}\right],\quad \text{for } \psi_1>0,\ \psi_2>0, \tag{10}$$

where $\mathrm{E_t}$ is the conditional expectations operator, $\rho\in(0,1)$ is a subjective discount factor, and $\chi\geqslant 0$ measures the relative risk aversion in labor, subject to the date-$s$ version of (7), (8), (9), as well as (5) (in the sense that, given the wage rate it charges for its labor service, the household always supplies its labor to meet the schedule of demand for its service as specified in (5)), for all $s\geqslant t$, where the household takes its initial capital and money holdings as given.

All households are price takers in the capital rental and money markets, as well as in the market for the composite final good, but they are monopolistic competitors in the labor markets where they set nominal wages for their labor services in a staggered fashion à là Calvo [17], with an identical and independent hazard rate $\theta$ of unable to adjusting their wages. With the large number of households, at each point in time there is a fraction $(1-\theta)$ of randomly selected households that can adjust wages. At a given date $t$, if a household $i$ gets the chance to set a new wage, then the optimal choice of nominal wage for its labor service subject to the demand schedule (5) is governed by the following wage-setting equation

$$W_t(i)=\left[\frac{\psi_2\epsilon}{\epsilon-1}\frac{W_t^{(1+\chi)\epsilon}n_t^{1+\chi}P_t+\mathrm{E_t}\sum_{s=t+1}^{\infty}(\theta\rho)^{s-t}(\prod_{h=1}^{s-t}\pi_{p,t+h})^{-1}W_s^{(1+\chi)\epsilon}n_s^{1+\chi}P_s}{W_t^{\epsilon}n_tc_t(i)^{-1}+\mathrm{E_t}\sum_{s=t+1}^{\infty}(\theta\rho)^{s-t}(\prod_{h=1}^{s-t}\pi_{p,t+h})^{-1}W_s^{\epsilon}n_sc_s(i)^{-1}}\right]^{\frac{1}{1+\chi\epsilon}}, \tag{11}$$

where $\pi_{p,t+h}\equiv P_{t+h}/P_{t+h-1}$ denotes the gross price inflation rate during period $t+h$ (for future reference, we will use $\pi_{w,t}\equiv W_t/W_{t-1}$ to denote the gross wage inflation rate during period $t$). On the other hand, regardless of whether or not the household gets a chance to set its wage, it can always choose its consumption, money and capital holdings, as well as the utilization rate for its capital in any given date. The resultant optimality conditions for these choices imply

$$r_t=u_t(i)^{\phi-1}, \tag{12}$$

$$\psi_1\frac{P_tc_t(i)}{M_t(i)}=1-\rho\mathrm{E_t}\left[\frac{P_tc_t(i)}{P_{t+1}c_{t+1}(i)}\right], \tag{13}$$

$$1=\rho\mathrm{E_t}\left\{\frac{c_t(i)}{c_{t+1}(i)}\left[1-\delta\big(u_{t+1}(i)\big)+r_{t+1}u_{t+1}(i)\right]\right\}, \tag{14}$$

where (12) comes from optimal capacity utilization choice, and (13) and (14) arise from optimal decisions for consumption and for money and capital holdings, respectively.

At each date $t$, a firm $j$ hires the composite labor service $n_t(j)$ from the distributors and capital $k_t(j)$ from the households (which is in the form of the composite final good) to produce a type $j$ intermediate good according to

$$y_t(j)=\left\{\left[u_t(j)k_t(j)\right]^{\alpha}n_t(j)^{1-\alpha}\right\}^{\eta},\quad \text{for } \alpha\in(0,1), \tag{15}$$

where $\eta>1$ measures the degree of increasing returns to scale at the individual firm level.

Firms are price takers in factor markets and monopolistic competitors in their output markets, where they set nominal prices for their goods in a synchronized fashion.[1] At a given date $t$, a firm $j$ chooses nominal price $P_t(j)$ for its product to maximize its period-$t$ profit

[1] It turns out that unsynchronized price-setting has a negligible effect on the local stability property of the model, so we consider synchronized price-setting, as in Benhabib and Farmer [13], to keep our analysis punchy.

$$P_t(j)y_t(j) - TC_t\big(j, y_t(j)\big), \tag{16}$$

taking the demand schedule (2) as given (in the sense that, given the price it charges for its product, the firm always supplies its output to meet the schedule of demand for its good as specified in (2)), where $TC_t(j, y_t(j))$ denotes the cost facing $j$ at $t$ for producing $y_t(j)$, resultant from choosing factor inputs to minimize $W_t n_t(j) + P_t r_t u_t(j) k_t(j)$ subject to (15). It can be verified that the profit maximization problem has an interior solution provided the degree of increasing returns to scale $\eta$ is bounded above by the monopolistic markup $\varepsilon$. The resulting factor demands by the firm are

$$n_t(j) = \left[\frac{(1-\alpha)r_t}{\alpha w_t}\right]^{\alpha} y_t(j)^{\frac{1}{\eta}}, \tag{17}$$

$$u_t(j)k_t(j) = \left[\frac{(1-\alpha)r_t}{\alpha w_t}\right]^{\alpha-1} y_t(j)^{\frac{1}{\eta}}, \tag{18}$$

where $w_t \equiv W_t/P_t$ denotes real wage rate for the composite labor service, and the optimal choice of its price as a markup of marginal cost satisfies

$$P_t(j) = \frac{\varepsilon}{\eta}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} P_t r_t^{\alpha} w_t^{1-\alpha} y_t(j)^{\frac{1-\eta}{\eta}}. \tag{19}$$

There is a monetary authority that follows a constant money supply growth rule and injects newly created money into the economy via the lump-sum transfers to the households,

$$\frac{M_t - M_{t-1}}{P_t} = \int_0^1 \tau_t(i)\,di, \tag{20}$$

where we will use $m_t = M_t/P_t$ to denote real money supply at date $t$.

Finally, market clearing requires $\int_0^1 y_t(i)\,di = y_t$ for the composite final good, $\int_0^1 n_t(j)\,dj = n_t$ for the composite labor, $\int_0^1 k_t(i)\,di = \int_0^1 k_t(j)\,dj$ $(\equiv k_t)$ for capital (in form of the composite good), $\int_0^1 u_t(i)k_t(i)\,di = \int_0^1 u_t(j)k_t(j)\,dj$ for capacity utilization, and $\int_0^1 M_t(i)\,di = M_t$ for money.

An equilibrium for this economy consists of allocations $y_t$ and $\{y_t(j)\}_{j\in[0,1]}$ for goods retailers, allocations $n_t$ and $\{n_t(i)\}_{i\in[0,1]}$ for worker distributors, allocations $y_t(i)$, $c_t(i)$, $k_{t+1}(i)$, $u_t(i)$, and $M_t(i)$, and wage $W_t(i)$, for household $i$, for all $i \in [0,1]$, allocations $k_t(j)$, $u_t(j)$, and $n_t(j)$, and price $P_t(j)$, for firm $j$, for all $j \in [0,1]$, together with price $P_t$ for the composite good, wage $W_t$ for the composite labor, and real rental rate $r_t$ for capital, that satisfy the following conditions: (i) given the prices and wages, respectively, the allocations for retailers and for distributors solve their cost minimization problems; (ii) given the price and wage indexes and capital rental rate, each household's allocations and wage solve its utility maximization problem, where the household supplies its differentiated labor to meet the demand schedule (5) given its chosen wage; (iii) given the price and wage indexes and capital rental rate, each firm's allocations and price solve its profit maximization problem, where the firm supplies its differentiated good to meet the demand schedule (2) given its chosen price; (iv) monetary policy is as specified; (v) markets for the composite final good, the composite labor, capital, capacity utilization, and money clear.

It is clear from (12) that optimal choice of capacity utilization rate is identical across households, $u_t(i) = r_t^{1/(\phi-1)} \equiv u_t$, thus we can drop the individual household index $i$ from this equation. The market clearing conditions for capital and for capacity utilization then imply $\int_0^1 u_t(j)k_t(j)\,dj = u_t k_t$. We can infer from (2), (3), and (19) that firms choose identical price, equal to the price level, and produce identical unit of output, equal to the quantity of the composite good. It then follows from (17) and (18) that optimal choices of the composite labor input and capacity utilization are identical across firms. In light of the market clearing conditions for the composite labor service and for capacity utilization, we conclude that an aggregate version of (17)–(19) also holds without the individual firm index $j$ attached. This gives rise to the following relations between aggregate quantities and real factor prices

$$w_t = \frac{(1-\alpha)\eta y_t}{\varepsilon n_t}, \qquad r_t = \frac{\alpha\eta y_t}{\varepsilon u_t k_t}, \tag{21}$$

and an aggregate version of production function,

$$y_t = \left[(u_t k_t)^\alpha n_t^{1-\alpha}\right]^\eta, \tag{22}$$

as in Benhabib and Farmer [13]. Finally, aggregating (7) across households, and using the market clearing condition for capital along with the fact that capacity utilization rate is identical across all households, we can rewrite the market clearing condition for the composite final good into an aggregate resource constraint,

$$y_t = c_t + k_{t+1} - \left[1-\delta(u_t)\right]k_t, \tag{23}$$

where $c_t$ denotes the aggregate consumption $\int_0^1 c_t(i)\,di$.

We assume that households have identical initial endowments of capital and money stock, own equal shares of firms' profits, and receive equal transfers from the government, and that there are (implicit) financial arrangements that make the households able to insure against idiosyncratic income risks that may arise from the unsynchronized wage adjustment, so that consumption is identical across the households, although their nominal wages and hours worked may differ from one another. This assumption of consumption insurance is standard in the literature on staggered wage-setting (e.g., Rotemberg and Woodford [50]; Erceg, Henderson, and Levin [26]; Christiano, Eichenbaum, and Evans [21]). With $c_t(i) = c_t$ for all $i \in [0, 1]$, money holding is identical across households in light of (13) and all wage-setting households choose identical wage in light of (11). Denote by $W_t^*$ the optimal wage chosen by a wage-setting household at date $t$ (i.e., the left hand side of (11)). We can then show that (6) reduces to

$$W_t = \left[\theta W_{t-1}^{1-\epsilon} + (1-\theta) W_t^{*\,1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{24}$$

It can be checked that there is a unique deterministic steady-state equilibrium for the economy. For local determinacy analysis, we can therefore examine a log-linearized system of equilibrium conditions around the steady state. Throughout the rest of the paper, we shall use a variable with a hat to denote the percentage deviation of that variable in level from its steady-state value.

We start with the following system that governs the log-linearized equilibrium dynamics for consumption, capital, labor, real balances, price inflation, real wage, and wage inflation,

$$\left[\frac{\theta(1+\chi\epsilon)}{(1-\theta)(1-\rho\theta)}\right](\widehat{\pi}_{w,t} - \rho \mathrm{E}_t\widehat{\pi}_{w,t+1}) = -k_c\widehat{k}_{t+1} + k_c\widehat{k}_t + (1+\chi)\widehat{n}_t, \tag{25}$$

$$\mathrm{E}_t\widehat{c}_{t+1} - (1-\rho)(\alpha\eta\tau_k - 1)\widehat{k}_{t+1} - (1-\rho)(1-\alpha)\eta\tau_n\mathrm{E}_t\widehat{n}_{t+1} = \widehat{c}_t, \tag{26}$$

$$k_c\widehat{k}_{t+1} = -\widehat{c}_t + (k_c + \alpha\eta\tau_k)\widehat{k}_t + (1-\alpha)\eta\tau_n\widehat{n}_t, \tag{27}$$

$$\rho\mathrm{E}_t\widehat{c}_{t+1} - \rho\mathrm{E}_t\widehat{m}_{t+1} = \widehat{c}_t - \widehat{m}_t, \tag{28}$$

$$\widehat{\pi}_{p,t} = \widehat{m}_{t-1} - \widehat{m}_t, \tag{29}$$

$$\widehat{w}_t = \alpha\eta\tau_k\widehat{k}_t + \big[(1-\alpha)\eta\tau_n - 1\big]\widehat{n}_t, \tag{30}$$

$$\widehat{\pi}_{w,t} = \widehat{\pi}_{p,t} + \widehat{w}_t - \widehat{w}_{t-1}, \tag{31}$$

where

$$k_c = \frac{\alpha\eta(\phi-1)\rho}{(\phi\varepsilon-\alpha\eta)(1-\rho)}, \qquad \tau_k = \frac{\phi-1}{\phi-\alpha\eta}, \qquad \tau_n = \frac{\phi}{\phi-\alpha\eta}.$$

Once these variables are obtained, the log-linearized equilibrium dynamics for real capital rental rate and capacity utilization rate can be determined from the log-linearized versions of (12) and (14) (with the individual household index $i$ dropped), and then the log-linearized equilibrium dynamics of real aggregate output can be determined from the log-linearized version of the aggregate production function (22), or the aggregate resource constraint (23). The local stability property of the model economy can thus be obtained by analyzing the system (25)–(31).

We first show that the stability property of the system (25)–(31) for the case with synchronized wage adjustment, that is, with a zero hazard rate $\theta$, can be obtained by analyzing a three-equation system. We first note that (25)–(28) in this case constitute a self-closed system for consumption, capital, labor, and real balances. Once these variables are obtained, price inflation can be determined from (29), real wage from (30), and then wage inflation from (31). We next notice using (27) that (25) in this case can be rewritten as $\widehat{c}_t - \alpha\eta\tau_k\widehat{k}_t + [1+\chi-(1-\alpha)\eta\tau_n]\widehat{n}_t = 0$. We can then use this relation to substitute out labor from (26) and (27), which together with (28) constitute a self-closed three-equation system in consumption, capital, and real balances,

$$\left[\frac{1+\chi-\rho(1-\alpha)\eta\tau_n}{1+\chi-(1-\alpha)\eta\tau_n}\right]\mathrm{E}_t\widehat{c}_{t+1} + (1-\rho)\left[1 - \frac{\alpha\eta(1+\chi)\tau_k}{1+\chi-(1-\alpha)\eta\tau_n}\right]\widehat{k}_{t+1} = \widehat{c}_t,$$

$$k_c\widehat{k}_{t+1} = -\left[\frac{1+\chi}{1+\chi-(1-\alpha)\eta\tau_n}\right]\widehat{c}_t + \left[k_c + \frac{\alpha\eta(1+\chi)\tau_k}{1+\chi-(1-\alpha)\eta\tau_n}\right]\widehat{k}_t,$$

$$\rho\mathrm{E}_t\widehat{c}_{t+1} - \rho\mathrm{E}_t\widehat{m}_{t+1} = \widehat{c}_t - \widehat{m}_t.$$

Since this system contains one predetermined variable (capital) and two jump variables (consumption and real balances), indeterminacy requires at least two eigenvalues lying inside the unit circle.

We now show that the stability property of the system (25)–(31) for the case with unsynchronized wage adjustment, that is, with a positive hazard rate $\theta$, can be obtained by analyzing a six-equation system. We first use (27) to substitute out $k_{t+1}$ from the right hand side of (25), we then use (30) to substitute out labor from (26) and (27), and we next use (29)–(31) to substitute out labor and wage inflation from (25). We arrive at the following system consisting of six equations,

$$\begin{aligned}\rho\mathrm{E}_t\widehat{m}_{t+1} - \rho\mathrm{E}_t\widehat{w}_{t+1} = {} & \gamma^{-1}\widehat{c}_t + \left[\frac{\gamma^{-1}\chi\alpha\eta\tau_k}{1-(1-\alpha)\eta\tau_n}\right]\widehat{k}_t + (1+\rho)\widehat{m}_t - \widehat{s}_t \\ & - \left[1+\rho+\gamma^{-1}\frac{1+\chi-(1-\alpha)\eta\tau_n}{1-(1-\alpha)\eta\tau_n}\right]\widehat{w}_t + \widehat{z}_t,\end{aligned}$$

$$\mathrm{E}_t\widehat{c}_{t+1} + (1-\rho)\left[1 - \frac{\alpha\eta\tau_k}{1-(1-\alpha)\eta\tau_n}\right]\widehat{k}_{t+1} + \left[\frac{(1-\rho)(1-\alpha)\eta\tau_n}{1-(1-\alpha)\eta\tau_n}\right]\mathrm{E}_t\widehat{w}_{t+1} = \widehat{c}_t,$$

$$k_c\widehat{k}_{t+1} = -\widehat{c}_t + \left[k_c + \frac{\alpha\eta\tau_k}{1-(1-\alpha)\eta\tau_n}\right]\widehat{k}_t - \left[\frac{(1-\alpha)\eta\tau_n}{1-(1-\alpha)\eta\tau_n}\right]\widehat{w}_t,$$

$$\rho \mathrm{E_t}\widehat{c}_{t+1} - \rho \mathrm{E_t}\widehat{m}_{t+1} = \widehat{c}_t - \widehat{m}_t,$$
$$\widehat{s}_{t+1} = \widehat{m}_t,$$
$$\widehat{z}_{t+1} = \widehat{w}_t,$$

where

$$\gamma = \frac{\theta(1+\chi\epsilon)}{(1-\theta)(1-\rho\theta)},$$

and $\widehat{k}_t, \widehat{s}_t, \widehat{z}_t$ are predetermined variables, and $\widehat{c}_t$, $\widehat{m}_t$, and $\widehat{w}_t$ are jump variables, so indeterminacy requires at least four eigenvalues lying inside the unit circle.

## 3. Calibration and indeterminacy results

A challenge facing the literature of equilibrium indeterminacy and sunspot-driven business cycle fluctuations based on increasing returns to scale in production is that the required degree of increasing returns for generating indeterminacy can be implausibly large and rise quickly with the relative risk aversion in labor. One point of this paper is that unsynchronized wage adjustment can both lower the degree of increasing returns to scale required for indeterminacy to an empirically plausible level and make it invariant to the relative risk aversion in labor.

### *3.1. Parameter calibration*

One period in our model corresponds to one quarter of a physical year. We set the quarterly discount factor $\rho$ to 0.99, which is consistent with a steady-state annualized real interest rate of 4 percent. Given this value of $\rho$, while having in mind a steady-state annual capital depreciation rate of 10 percent, we calibrate the value of $\phi$ to 1.3885 to be consistent with a steady-state quarterly capital depreciation rate of 2.6 percent in the light of the steady-state equilibrium relationship $\phi = [1-\rho(1-\delta)]/(\rho\delta)$.[2]

To consider a value for $\chi$, the relative risk aversion in labor, we note that its inverse corresponds to the Frisch elasticity of labor supply. A large number of empirical studies have estimated the labor supply elasticity. The evidence obtained based on different sources of data, frequencies of time, sample periods, aggregation levels, substitution margins (intensive versus extensive), seasonality adjustments, and estimation procedures suggests that the elasticity lies between 0.05 and 2. Many of these studies based on life-cycle data of hours worked (intensive margin) by men at annual or lower frequencies find an elasticity in the range of 0.05 to 0.3, as summarized by Pencavel [46]. Evidence on a low elasticity is also found by Altonji [2], Ball [3], and Card [18]. Using monthly data from the Denver Income Maintenance Experiment, MaCurdy [42] finds that the (intensive) elasticity can be above 0.3 though it is below 0.7. Rupert, Rogerson, and Wright [51] obtain similar estimates taking home production into account. Using tri-annual micro panel data of the Survey of Income and Program Participation, Kimmel and Kniesner [37] estimate an hours worked elasticity (intensive margin) of 0.5 and an employment elasticity (extensive margin) of 1.5. Mulligan [44] shows that the elasticity can lie in the 1–2 range. Many other of these studies, using multi-industry panel or macro-level data (as reported by Browning, Hansen, and Heckman [16]), using industry-specific data (e.g., Treble [58];

[2] The quarterly and annual capital depreciation rates in the steady state are related by $0.026 = 1-(1-0.1)^{1/4}$.

Carrington [19]; Oettinger [45]), and using data at several different frequencies (e.g., Abowd and Card [1]) or taking into account seasonal variation in total hours worked (e.g., Barsky and Miron [5]; Hall [30]), also provide corroborating evidence that helps reach a consensus that the elasticity lies between 0.05 and 2. This suggests a value for $\chi$ between 0.5 and 20. As we will show below, our indeterminacy result is virtually invariant to the choice of $\chi$ within a broad range.

Turning to the elasticity of substitution between differentiated labor skills, we set $\epsilon$ to 4, which is the mid point of the empirical estimates by Griffin [28,29] that range from 2 to 6. Our results are not sensitive to alternative values of $\epsilon$ within this empirically plausible range.

To assign a value to the hazard rate of wage adjustment $\theta$, we note that $(1-\theta)$ corresponds to the quarterly frequency of wage adjustment and the average duration of newly set wages for individual workers is given by $1/(1-\theta)$. There is a large body of empirical work that has estimated the average length of wage-setting interval, the frequency of wage adjustment, or the hazard rate. The evidence obtained based on different sources of data, sample periods, frequencies of time, aggregation levels, and estimation procedures reveals a relevant range for $\theta$ between 0.42 and 0.86. Many studies in the 1980s and the 1990s suggest an average length of wage-setting interval of about four quarters or longer for the United States and other countries, such as Canada and Sweden, based on direct microeconomic evidence on wage-setting procedures and indirect estimation from macroeconomic data. Taylor [57] contains a comprehensive survey of this literature (see also Kahn [36]). This corresponds to a value for $\theta$ of at least 0.75. The more recent literature is also divided into two strands, one based on macro data and the other on micro data. The macro studies typically use SDGE models to estimate the hazard rate along with other structural parameters. While the estimation of $\theta$ by Christiano, Eichenbaum, and Evans [21] for the US ranges from 0.42 to 0.80, depending on alternative model specifications, Christiano, Motto, and Rostagno [22] report their estimate of $\theta$ at 0.80 for the US and 0.83 for the Euro Area. The Bayesian estimate of $\theta$ obtained by Levin, Onatski, Williams, and Williams [38] based on postwar US data ranges from 0.72 to 0.85. Also applying Bayesian methods, Smets and Wouters [54] estimate $\theta$ between 0.662 and 0.824 for the Euro Area, and Smets and Wouters [55] estimate $\theta$ between 0.60 and 0.81 for the US. DiCecio [23] and Matheron and Poilly [43] obtain similar estimates of $\theta$ for the US economy (0.73 and 0.775 respectively). The recent micro studies focus on estimating the frequency of wage adjustment. Using quarterly firm-level data from the Survey on Labor Activity and Employment Status (ACEMO) carried out by the French Ministry of Labor and Social Affairs for French firms over the period from 1998 to 2005, Heckel, Le Bihan, and Montornès [31] estimate a quarterly frequency of wage adjustment of about 35 percent, implying a value for the hazard rate $\theta$ of 0.65. Analyzing data from the last two complete panels of the Survey of Income and Program Participation (SIPP) (1996–1999 and 2001–2003) run by the Bureau of Labor Statistics (BLS), Barattieri, Basu, and Gottschalk [4] find that, in an average quarter, the probability that a representative American will experience a nominal wage change is between 14 and 16 percent, implying a value of $\theta$ between 0.84 and 0.86. Using an administrative dataset of monthly frequency that covers all firms and employees in Luxembourg over the period between January 2001 and December 2006, and correcting for measurement errors and adjusting for institutional factors, Lunnemann and Wintr [39] estimate a monthly frequency of wage adjustment between 5 and 7 percent, implying a quarterly frequency of wage adjustment between 14 and 19 percent. This gives rise to a range for $\theta$ between 0.81 and 0.86. In light of the empirical evidence, we set the hazard rate of wage adjustment $\theta$ to 0.75 in the baseline model, as is standard in most theoretical studies of unsynchronized wage adjustment (e.g., Erceg, Henderson, and Levin [26]; Rabanal and Rubio-Ramírez [47]; Sveen and Weinke [56]; Casares [20]).

We will also show that our results are robust to alternative values of $\theta$ within its empirically plausible range.

We here recall that the profit maximization problem has a well-defined interior solution as long as the degree of increasing returns to scale $\eta$ is bounded above by the monopolistic markup $\varepsilon$. In light of the studies by Domowitz, Hubbard, and Petersen [25], Shapiro [52], Basu [6], Rotemberg [48], Rotemberg and Woodford [50], Basu and Fernald [7–11], and Basu and Kimball [12], a value of $\varepsilon$ between 1.05 and 1.15 can be empirically plausible. This provides a useful criterion for assessing whether the required increasing returns to scale for generating indeterminacy is empirically reasonable. In searching for the values of $\eta$ that produce indeterminacy, we set the two parameters $\eta$ and $\varepsilon$ equal so that profits for firms are zero.[3] With zero economic profit, the parameter $\alpha$ corresponds to the share of payments to capital in total value added in the National Income and Product Account. The implied value of $\alpha$ is about 0.3, in line with the value used in the standard business cycle literature.

Note that the two parameters $\psi_1$ and $\psi_2$ in the utility function do not affect the log-linearized equilibrium dynamics and thus we do not need to assign any particular values to them.

### 3.2. *Unsynchronized wage adjustment reduces the required degree of increasing returns for indeterminacy*

To appreciate the role of unsynchronized wage adjustment in reducing the required degree of increasing returns for indeterminacy, it is informative to contrast our results obtained under unsynchronized wage adjustment and under synchronized wage adjustment. Here it is useful to note that, with synchronized wage adjustment, that is, with $\theta = 0$, the optimal wage-setting equation (11) (with the individual household index $i$ dropped) and its log-linearized version (25) simplify to $w_t = \psi_2 \epsilon (\epsilon - 1)^{-1} c_t n_t^{\chi}$ and $0 = -k_c \widehat{k}_{t+1} + k_c \widehat{k}_t + (1+\chi)\widehat{n}_t$, respectively, while all the other equilibrium conditions remain the same. This shows that our model featuring differentiated skills and monopolistic competition in the labor market with a zero hazard rate of wage adjustment is first-order equivalent to an otherwise identical model but with a homogenous skill and perfect competition in the labor market, which can be cast as an externality model such as Wen [59] or Benhabib and Wen [15] with money in the utility function. The comparison thus helps put into perspective our results in the light of the general literature on indeterminacy based on increasing returns to scale in production.

Table 2 reports the minimal degree of increasing returns to scale that can generate indeterminacy for the case with synchronized wage adjustment and the case with unsynchronized wage adjustment, for three values of $\chi$, the calibrated value 1 and two additional values 0.5 and 2 in its empirically plausible range. All the other parameters take on their calibrated values reported in Table 1. The middle column of the table shows the results under synchronized wage adjustment and the three corresponding values of $\eta$, 1.8, 1.5, and 2.3, are all beyond its empirically plausible range [1.05, 1.15]. The right column shows the results under unsynchronized wage adjustment and the corresponding value of $\eta$ is reduced to 1.1, right in the middle of its empirically plausible range, for all the three values of $\chi$. A message from the table is thus that unsynchronized wage adjustment can lower the required degree of increasing returns for indeterminacy to an empirically plausible level.

[3] The equilibrium profit-output ratio in our model is equal to $1 - \eta/\varepsilon$. Much empirical evidence suggests that profit rates are close to zero (e.g., Rotemberg and Woodford [49]; Basu and Fernald [9]). We have also experimented with the cases of positive profit rates and find that our results do not change.

Table 1
Calibration-parameter values.

| Parameters | Values |
|---|---|
| Subjective quarterly discount factor ($\rho$) | 0.99 |
| Curvature parameter in the capital depreciation function ($\phi$) | 1.3885 |
| Elasticity of substitution of differentiated labor skills ($\epsilon$) | 4 |
| Hazard rate of wage adjustment ($\theta$) | 0.75 |
| Share of payments to capital in total value added ($\alpha$) | 0.3 |

Table 2
Minimal degree of increasing returns to scale that can generate indeterminacy.

| Relative risk aversion in labor | Synchronized wage adjustment | Unsynchronized wage adjustment |
|---|---|---|
| $\chi = 0.5$ | 1.5 | 1.1 |
| $\chi = 1^\star$ | 1.8 | 1.1 |
| $\chi = 2$ | 2.3 | 1.1 |

*Note*. The value of $\chi$ denoted by $\star$ in the first column of the table is the baseline calibration for $\chi$. All other parameters take on their calibrated values.

### 3.3. *Unsynchronized wage adjustment makes the required degree of increasing returns for indeterminacy invariant to the relative risk aversion in labor*

The numbers presented in Table 2 also reveal that the required degree of increasing returns for indeterminacy can rise significantly with $\chi$ under synchronized wage adjustment while it seems invariant to $\chi$ under unsynchronized wage adjustment. We find that this is true in general.

To put this contrast into perspective, Fig. 1 plots the minimal value of $\eta$ that can generate indeterminacy for the case with synchronized wage adjustment and the case with unsynchronized wage adjustment for many more values of $\chi$, ranging from 0 to 20. Again, all the other parameters take on their calibrated values reported in Table 1. The broken line in the figure corresponds to the case with synchronized wage adjustment and, indeed, it increases rapidly with $\chi$: it takes on a value of 1.1 when $\chi$ is set to 0, quickly rises to 1.2 as $\chi$ edges up to 0.1, and continues to increase with $\chi$ at a fast pace. The solid line corresponds to the case with unsynchronized wage adjustment and, in sharp contrast to the broken line, it stays flat at 1.1 regardless of the value of $\chi$.[4]

As a further robustness check, Fig. 2 plots the minimal degree of increasing returns that can generate indeterminacy as a function of the relative risk aversion in labor under synchronized wage adjustment, as well as under unsynchronized wage adjustment with three alternative hazard rates, while all other parameters take on their calibrated values.[5] As can be seen from the figure,

[4] Notice that the two lines in Fig. 1 intercept with the vertical axis at the same point. This is so since with zero relative risk aversion in labor, households would not mind varying their labor hours even with a tiny change in wages so equilibrium wages would not change much with employment even under synchronized wage adjustment, and the effect of unsynchronized wage adjustment is muted. We have obtained an analytical result which shows that the local stability condition is the same under synchronized wage adjustment and under unsynchronized wage adjustment if $\chi$ is set to 0. This result is available from the authors upon request.

[5] In our quarterly model, the benchmark calibration of the hazard rate of wage adjustment, $\theta = 0.75$, implies that wages are adjusted once every four quarters on average, and a value of $\theta$ equal to 0.50 implies that wages are adjusted every other quarter on average, while as a value of $\theta$ equal to 0 implies that wages are adjusted every quarter, corresponding

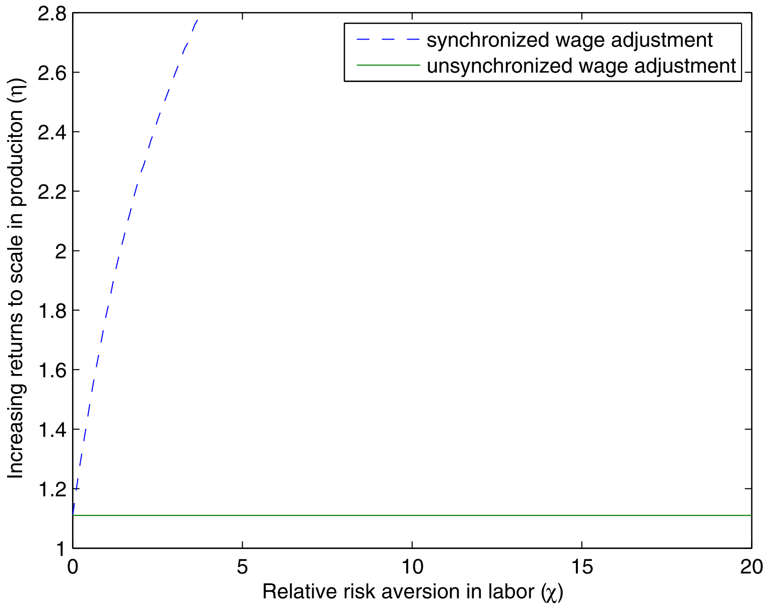


Fig. 1. Minimal degree of increasing returns to scale that can generate indeterminacy as a function of the relative risk aversion in labor (all other parameters take on their calibrated values).

our results are robust to alternative values for the hazard rate of wage adjustment within its empirically plausible range.

Putting the above results together we conclude that, not only does unsynchronized wage adjustment lower the required degree of increasing returns for indeterminacy to an empirically plausible level, it also makes it invariant to the relative risk aversion in labor. As a consequence, an indeterminate equilibrium and thus sunspot-driven business cycle fluctuations may emerge for empirically plausible increasing returns regardless of the value of the relative risk aversion in labor.

## 4. Some intuition

To understand why unsynchronized wage adjustment makes the required increasing returns to scale for indeterminacy smaller and its sensitivity to the relative risk aversion in labor disappear, we first note that the only difference between the case with synchronized wage adjustment and the case with unsynchronized wage adjustment lies in the left hand side of (25), derived from the log-linearized versions of (11) and (24): it is zero under synchronized wage adjustment but generally nonzero under unsynchronized wage adjustment. We next recall that increasing returns to scale in production (e.g., convexity) makes indeterminacy more likely to occur because it makes the optimal response of labor to a contemplated increase in capital stronger,

to the case with synchronized (and flexible) wage adjustment. While the result under $\theta = 0.42$ (the lower bound on the empirical estimates of the hazard rate of wage adjustment) is very similar to that under $\theta = 0.50$ (the broken line with dots in Fig. 2), the result under $\theta = 0.86$ (the upper bound on the empirical estimates of the hazard rate of wage adjustment) is very similar to that under $\theta = 0.80$ (the broken line in Fig. 2).

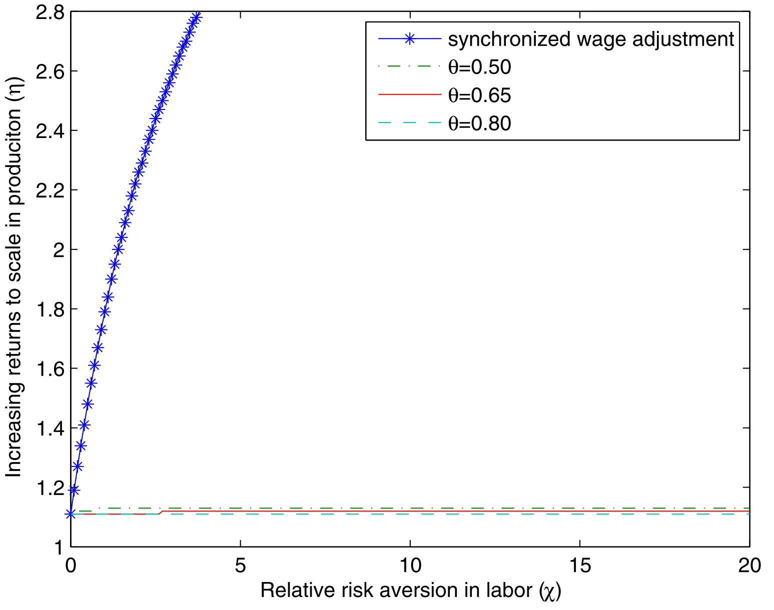


Fig. 2. Minimal degree of increasing returns to scale that can generate indeterminacy as a function of the relative risk aversion in labor (with three alternative hazard rates for unsynchronized wage adjustment while all other parameters take on their calibrated values).

resulting in a smaller decrease or even increase in the marginal product of capital and thus the contemplated increase in capital is more likely to be self-fulfilling. That said, with synchronized wage adjustment, a large $\chi$ serves as a counter-force to dampen the response of labor, as a large relative risk aversion in labor makes it more costly to raise labor hours under synchronized wage adjustment. This can be seen by imposing $\theta = 0$ in (25): the response of $\widehat{n}_t$ to a contemplated increase in $\widehat{k}_{t+1}$ (note that $\widehat{k}_t$ is predetermined) is then given by $k_c/(1+\chi)$, which clearly decreases with $\chi$. Thus, a large relative risk aversion in labor requires a large degree of increasing returns to scale in production to start with in order to generate indeterminacy. In fact, we obtain a necessary condition for indeterminacy under synchronized wage adjustment as

$$\eta > \frac{\phi(1+\chi)}{\alpha(1+\chi)+(1-\alpha)\phi\rho},$$

which puts a lower bound on the required degree of increasing returns and the lower bound increases with $\chi$, manifesting our results in Section 3 for the case with synchronized wage adjustment.

Unsynchronized wage adjustment effectively dampens the cost of raising labor hours and relieves the counter-force brought about by the relative risk aversion in labor. With unsynchronized wage adjustment, firms can optimally increase employment in response to a contemplated increase in capital without equilibrium wages rising as they would with synchronized wage adjustment. The fact that labor can be more elastic to capital if $\theta > 0$ can be seen from (25): a change on the right hand side due to a large response of labor to capital can be well matched by a change on the left hand side now. To understand why $\chi$ multiplied by $\epsilon$ enters the left hand side of (25) we note that, with unsynchronized wage adjustment, even households that can adjust their wages will choose not to increase their wages much, as firms can substitute away from their more expensive labor services and toward the cheaper ones provided by other households that can't change their wages. The strength of this deterrence depends on how easy it is to substitute across the differentiated labor skills, as measured by $\epsilon$, and on how fast marginal disutility of working changes as hours worked fall, as determined by $\chi$. This substitution across differentiated labor skills following a relative wage change results in a small overall wage increase and a large overall employment response to the contemplated increase in capital. With labor being more elastic to capital, the marginal product of capital does not decrease as fast or even increases with capital and a self-fulfilling equilibrium is more likely to occur. The presence of $\chi$ on the left hand side of the equation via a positive $\theta$ essentially cancels out its effect on the right hand side, making the required increasing returns to scale for indeterminacy invariant to $\chi$.

The micro foundation for a *relative wage effect* arising from unsynchronized wage adjustment under monopolistic competition to give rise to more elastic responses of aggregate labor to aggregate demand shocks was established by Huang and Liu [34]. The above discussion adapted the insight to the situation with a sunspot shock in our convex production economy. To put it into perspective, we first note that *no* such relative wage effect exists under synchronized wage adjustment, since in this case all households choose the same wage (so relative wage is always unitary) and work for the same number of hours at a given date. In response to a contemplated increase in capital and production, the households require an increase in their wage rate in order for them to be willing to supply an increased amount of labor to work with the contemplated increase in capital. Their required increase in wage rate is greater, the greater is their relative risk aversion in labor $\chi$, as a greater $\chi$ implies that the households are more averse to the increment in their hours worked and thus need to be compensated for by a higher wage rate. Hence, in order for the contemplated increase in capital to be self-fulfilling, a greater degree of increasing returns to scale in production is needed, the greater is $\chi$, to deter the required higher wage from giving rise to a sharply increasing marginal production cost.

We then note that this effect of the relative risk aversion in labor $\chi$ on the required degree of convexity for indeterminacy is in place whether wage adjustment is synchronized or unsynchronized.[6] Thus, under unsynchronized wage adjustment, in response to a contemplated increase in capital and production, a household that gets the chance to adjust its wage would still have an incentive to set a higher wage, the greater is $\chi$, for the reason stated above. However, given that a fraction of households do not get the chance to adjust their wages, an increase in the wage of an adjusting household would also mean an increase in its wage relative to the wages of the non-adjusting households. Firms then would substitute away from the relatively more expensive labor services of the adjusting households toward the relatively cheaper labor services of the non-adjusting households, whose marginal disutilities of working then would rise and marginal utilities of income fall. Thus, as soon as the latter group of households get a chance to adjust their wages, they would raise their wages to re-balance between their marginal disutilities of working and their marginal utilities of income. At this point, however, the former group of households that adjusted their wages last time may not get another chance to reset their wages again, so their wages would fall relative to the wages of current adjusters, and then

[6] This is reflected by the fact that the dependence of the right hand side of (25) on $\chi$ is independent of the hazard rate of unable to adjusting wage $\theta$.

firms would substitute away from the labor services of the current adjusters toward their labor services. The greater is their relative risk aversion in labor, the greater are the incentives of households to avoid such excessive fluctuations in their hours worked over time associated with the relative wage effect that would arise from wage dispersions across the households at various points in time. This implies that, in response to a contemplated increase in capital and production, a household that gets the chance to adjust its wage actually has a greater incentive to keep its wage in line with its peers' in the first place, the greater is $\chi$, as far as the relative wage consideration goes. This goes against the positive effect of $\chi$ on the incentive to set a higher wage discussed above.[7] Strategic complementarity in wage-setting implies that all adjusting households choose not to raise their wages by much even when they have a chance to do so. Thus endogenous wage inertia arises regardless of the value of $\chi$. This means that firms can increase employment with smaller increases in equilibrium wages under unsynchronized wage adjustment than under synchronized wage adjustment,[8] and so a smaller degree of increasing returns to scale in production suffices to generate a flat or even decreasing marginal cost to make the contemplated increase in capital self-fulfilling, independent of the value of $\chi$.

To drive home our intuition in a more rigorous way, we can use (26) and (28)–(31) to solve for the wage inflation rates on the left hand side of (25) in terms of labor and capital. We consider $\eta$ in the range $(\rho^{-1}, [\rho(1-\alpha)]^{-1})$, set $\phi = \alpha\eta[1-\rho(1-\alpha)\eta]^{-1}$, and impose perfect foresight to simplify exposition. We note that $\phi$ so set is greater than $\rho^{-1}$ and the implied steady-state capital depreciation rate is strictly between 0 and 1 for any value of $\eta$ in the above range; and as $\eta$ varies across this range, $\phi$ varies from arbitrarily close to $\rho^{-1}$ to arbitrarily close to infinity. We also note that the lower end of this range for $\eta$ is close to 1 for a value of $\rho$ close to 1.[9] With some algebra, we obtain the following simplified version of (25):

$$\left[\frac{\theta(1+\chi\epsilon)}{(1-\theta)(1-\rho\theta)}\right]\left[-k_w\widehat{k}_{t+1}+\widehat{q}_{t-1}+(1-\rho)\widehat{n}_t\right] = -k_c\widehat{k}_{t+1}+k_c\widehat{k}_t+(1+\chi)\widehat{n}_t, \quad (32)$$

where

$$k_w = \frac{\rho(\eta-1)}{(1-\alpha)\eta},$$

and $\widehat{q}_{t-1}$ is predetermined and is given by

$$\widehat{q}_{t-1} = \left[\frac{1}{\rho} - 1 - \frac{\eta-1}{\rho(1-\alpha)\eta}\right]\widehat{k}_{t-1} + \left[\frac{2(\eta-1)}{(1-\alpha)\eta} - 1 + \rho\right]\widehat{k}_t + \left(1-\frac{1}{\rho}\right)\widehat{n}_{t-1}.$$

Eq. (32) helps make our intuition more transparent. Since $k_w$ increases with $\eta$ and $\rho < 1$, unsynchronized wage adjustment makes given existing returns more effective in generating an elastic response of labor to a contemplated increase in capital, as a change on the right hand side

[7] As can be seen from the left hand side of (25), this counter effect is in place as long as wage adjustment is unsynchronized (i.e., $\theta > 0$) and the differentiated labor skills are substitutable to some extent (i.e., $\epsilon > 0$); and, the counter effect is greater, the greater are $\theta$ and $\epsilon$.

[8] Here it is useful to recall that, as in the standard models of monopolistic competition in the labor markets à là Dixit–Stiglitz [24], imperfectly competitive households choose the wages for their differentiated labor skills and supply their labor services to meet the demand by firms at their chosen wages.

[9] For example, for our calibrated values $\rho = 0.99$ and $\alpha = 0.3$, this range for $\eta$ is (1.01, 1.57).

of (32) can be matched by a change on the left hand side due to a large increase in labor, and the response is no longer sensitive to $\chi$, as its effects on both sides of the equation are essentially canceled out.

## 5. Computing sunspots equilibria

To gain further insight into the dynamic properties of the model, we study the model's general solution and impulse responses to various shocks. We consider two fundamental shocks as in Benhabib and Wen [15], a preference shock and a government spending shock. As in Wen [59] and Benhabib and Wen [15], period utility derived from date-$t$ consumption is now specified as $\log(c_t - \mu_t)$, where $\mu_t$ is a shock to consumption that generates the urge to consume. We also consider as in Benhabib and Wen [15] a shock to government spending, $g_t$, representing a pure resource drain from the economy. The government budget constraint in period $t$ is now specified as $(M_t - M_{t-1})/P_t = \int_0^1 \tau_t(i)\,di + g_t$, and the resource constraint for period $t$ is now given by $y_t = c_t + k_{t+1} - [1 - \delta(u_t)]k_t + g_t$. We postulate stationary AR(1) log-normal processes for the fundamental shocks,

$$\log(\mu_t) = (1 - \rho_\mu)\log(\mu) + \rho_\mu \log(\mu_{t-1}) + \varepsilon_{\mu t},$$
$$\log(g_t) = (1 - \rho_g)\log(g) + \rho_g \log(g_{t-1}) + \varepsilon_{gt},$$

where $\varepsilon_{\mu t}$ and $\varepsilon_{gt}$ are independent and identically distributed white-noise innovations, with finite standard deviations, $\sigma_\mu$ and $\sigma_g$, respectively.

The log-linearized equilibrium system is then given by

$$\gamma(\widehat{\pi}_{w,t} - \rho \mathrm{E_t}\widehat{\pi}_{w,t+1}) = \mu_c\widehat{c}_t + \chi\widehat{n}_t - \widehat{w}_t + (1-\mu_c)\widehat{\mu}_t, \tag{33}$$

$$\mu_c \mathrm{E_t}\widehat{c}_{t+1} - (1-\rho)(\alpha\eta\tau_k - 1)\widehat{k}_{t+1} - (1-\rho)(1-\alpha)\eta\tau_n \mathrm{E_t}\widehat{n}_{t+1} = \mu_c\widehat{c}_t + (1-\mu_c)(1-\rho_\mu)\widehat{\mu}_t, \tag{34}$$

$$k_c\widehat{k}_{t+1} = -(1-g_y)\widehat{c}_t + (k_c + \alpha\eta\tau_k)\widehat{k}_t + (1-\alpha)\eta\tau_n\widehat{n}_t - g_y\widehat{g}_t, \tag{35}$$

$$\mu_c\widehat{c}_t - \rho\mu_c\mathrm{E_t}\widehat{c}_{t+1} + (1-\mu_c)(1-\rho\rho_\mu)\widehat{\mu}_t = (1-\rho)\widehat{m}_t + \rho\mathrm{E_t}\widehat{\pi}_{p,t+1}, \tag{36}$$

$$\widehat{\pi}_{p,t} = \widehat{m}_{t-1} - \widehat{m}_t, \tag{37}$$

$$\widehat{w}_t = \alpha\eta\tau_k\widehat{k}_t + \left[(1-\alpha)\eta\tau_n - 1\right]\widehat{n}_t, \tag{38}$$

$$\widehat{\pi}_{w,t} = \widehat{\pi}_{p,t} + \widehat{w}_t - \widehat{w}_{t-1}, \tag{39}$$

$$\widehat{y}_t = \widehat{w}_t + \widehat{n}_t, \tag{40}$$

$$\widehat{i}_t = \frac{\rho(\phi-1)}{1-\rho}(\widehat{k}_{t+1} - \widehat{k}_t) + \widehat{y}_t, \tag{41}$$

$$\phi\widehat{u}_t = \widehat{y}_t - \widehat{k}_t, \tag{42}$$

$$\widehat{\mu}_t = \rho_\mu\widehat{\mu}_{t-1} + \varepsilon_{\mu t}, \tag{43}$$

$$\widehat{g}_t = \rho_g\widehat{g}_{t-1} + \varepsilon_{gt}, \tag{44}$$

where

$$\mu_c = \frac{1}{1-\mu/c}, \qquad g_y = \frac{\phi\varepsilon}{\phi\varepsilon - \alpha\eta}\frac{g}{y},$$

and we have invoked the condition that $\mathrm{E_t}\widehat{k}_{t+1} = \widehat{k}_{t+1}$, since capital at date $t+1$ is known at date $t$ so the one-period forecast error on capital is zero, and that $\mathrm{E_t}\widehat{\mu}_{t+1} = \rho_\mu \widehat{\mu}_t$ and $\mathrm{E_t}\widehat{g}_{t+1} = \rho_g \widehat{g}_t$, since agents know the probability distributions of the fundamental shocks so the forecast errors on these shocks are equal to the true disturbances.

We will use an algorithm proposed by Sims [53], Lubik and Schorfheide [40], and Hespeler [32,33] to solve the log-linearized equilibrium system. To this end, we introduce endogenous forecast errors $\eta_t^c$, $\eta_t^n$, $\eta_t^{\pi_w}$, and $\eta_t^{\pi_p}$ as $\eta_t^c = \widehat{c}_t - \mathrm{E}_{t-1}\widehat{c}_t$, $\eta_t^n = \widehat{n}_t - \mathrm{E}_{t-1}\widehat{n}_t$, $\eta_t^{\pi_w} = \widehat{\pi}_{w,t} - \mathrm{E}_{t-1}\widehat{\pi}_{w,t}$, and $\eta_t^{\pi_p} = \widehat{\pi}_{p,t} - \mathrm{E}_{t-1}\widehat{\pi}_{p,t}$, and we denote $\xi_t^c = \mathrm{E_t}\widehat{c}_{t+1}$, $\xi_t^n = \mathrm{E_t}\widehat{n}_{t+1}$, $\xi_t^{\pi_w} = \mathrm{E_t}\widehat{\pi}_{w,t+1}$, and $\xi_t^{\pi_p} = \mathrm{E_t}\widehat{\pi}_{p,t+1}$. The system can then be represented by

$$\Gamma_0 x_t = \Gamma_1 x_{t-1} + \Psi \varepsilon_t + \Pi \eta_t, \tag{45}$$

where

$$x_t = \begin{bmatrix} \widehat{y}_t & \widehat{c}_t & \widehat{n}_t & \widehat{i}_t & \widehat{k}_{t+1} & \widehat{\pi}_{w,t} & \widehat{\pi}_{p,t} & \widehat{w}_t & \widehat{m}_t & \widehat{u}_t & \xi_t^c & \xi_t^n & \xi_t^{\pi_w} & \xi_t^{\pi_p} & \widehat{\mu}_t & \widehat{g}_t \end{bmatrix}',$$

$$\varepsilon_t = [\varepsilon_{\mu t} \quad \varepsilon_{gt}]', \qquad \eta_t = \begin{bmatrix} \eta_t^c & \eta_t^n & \eta_t^{\pi_w} & \eta_t^{\pi_p} \end{bmatrix}',$$

and the two $16 \times 16$ matrices, $\Gamma_0$ and $\Gamma_1$, the $16 \times 2$ matrix, $\Psi$, and the $16 \times 4$ matrix, $\Pi$, are defined as follows:

$$\Gamma_0 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\gamma\rho & 0 & \mu_c - 1 & 0 \\
0 & 0 & 0 & 0 & \frac{\alpha\eta\tau_k - 1}{(\rho-1)^{-1}} & 0 & 0 & 0 & 0 & 0 & \mu_c & \frac{(1-\alpha)\eta\tau_n}{(\rho-1)^{-1}} & 0 & 0 & \frac{1-\rho_\mu}{(\mu_c-1)^{-1}} & 0 \\
0 & 0 & 0 & 0 & k_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g_y \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\rho & 0 & \rho\mu_c & 0 & 0 & \rho & \frac{1-\rho\rho_\mu}{(\mu_c-1)^{-1}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & \frac{\rho(\phi-1)}{1-\rho} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\phi & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix},$$

$$\Gamma_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_c & \chi & -\gamma & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_c & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_c + \alpha\eta\tau_k & 0 & 0 & 0 & 0 & 0 & g_y - 1 & (1-\alpha)\eta\tau_n & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_c & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & \alpha\eta\tau_k & 0 & 0 & 0 & 0 & 0 & 0 & (1-\alpha)\eta\tau_n - 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\rho(\phi-1)}{1-\rho} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_\mu & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_g
\end{bmatrix},$$

$$\Psi = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \Pi = \begin{bmatrix} \mu_c & \chi & -\gamma & 0 \\ \mu_c & 0 & 0 & 0 \\ g_y - 1 & (1-\alpha)\eta\tau_n & 0 & 0 \\ \mu_c & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & (1-\alpha)\eta\tau_n - 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The system can be transformed (and sorted and partitioned) through a generalized complex Schur decomposition of $\Gamma_0$ and $\Gamma_1$ by four $16 \times 16$ matrices, $Q$, $Z$, $\Lambda$, and $\Omega$, with $\Lambda$ and $\Omega$ being upper triangular, $Q'\Lambda Z' = \Gamma_0$, $Q'\Omega Z' = \Gamma_1$, and $QQ' = ZZ' = I$, into a system of 16 equations about transformed variables $\omega_t = Z'x_t$,

$$\left[\begin{array}{c|c} \underset{(16-e)\times(16-e)}{\Lambda_{11}} & \underset{(16-e)\times e}{\Lambda_{12}} \\ \hline \underset{e\times(16-e)}{0} & \underset{e\times e}{\Lambda_{22}} \end{array}\right] \cdot \omega_t = \left[\begin{array}{c|c} \underset{(16-e)\times(16-e)}{\Omega_{11}} & \underset{(16-e)\times e}{\Omega_{12}} \\ \hline \underset{e\times(16-e)}{0} & \underset{e\times e}{\Omega_{22}} \end{array}\right] \cdot \omega_{t-1} + \left[\begin{array}{c} \underset{(16-e)\times 16}{Q_1} \\ \hline \underset{e\times 16}{Q_2} \end{array}\right] \cdot (\Psi\varepsilon_t + \Pi\eta_t),$$

in which the last $e \in [0, 16]$ equations are explosive and the first $(16 - e)$ equations are stable. Since the rows of the $e \times 4$ matrix $Q_2\Pi$ are potentially linearly dependent, its singular value decomposition can generally be expressed as

$$\underset{e\times 4}{Q_2\Pi} = \underset{e\times e}{U} \cdot \underset{e\times 4}{D} \cdot \underset{4\times 4}{V'} = \left[\begin{array}{c|c} \underset{e\times s}{U_1} & \underset{e\times(e-s)}{U_2} \end{array}\right] \left[\begin{array}{c|c} \underset{s\times s}{D_{11}} & \underset{s\times(4-s)}{0} \\ \hline \underset{(e-s)\times s}{0} & \underset{(e-s)\times(4-s)}{0} \end{array}\right] \left[\begin{array}{c} \underset{s\times 4}{V_1'} \\ \hline \underset{(4-s)\times 4}{V_2'} \end{array}\right]$$
$$= \underset{e\times s}{U_1} \cdot \underset{s\times s}{D_{11}} \cdot \underset{s\times 4}{V_1'},$$

where $D_{11}$ is an invertible, diagonal matrix, and $U$ and $V$ are orthonormal matrices.

When the degree of returns to scale is smaller than 1.1, we have $e = s = 4$, and the system has a unique equilibrium in which the expectations errors $\eta_t$ are uniquely determined by the fundamental shocks $\varepsilon_t$,

$$\eta_t = -V_1 D_{11}^{-1} U_1' Q_2 \Psi \varepsilon_t, \tag{46}$$

and the solution satisfies

$$\Gamma_0 x_t = \Gamma_1 x_{t-1} + \left(I - \Pi V_1 D_{11}^{-1} U_1' Q_2\right)\Psi\varepsilon_t. \tag{47}$$

Decompose $Z$ into $Z = [\underset{16\times 12}{Z_1} \ \underset{16\times 4}{Z_2}]$, the unique solution under determinacy is given by

$$\begin{aligned} x_t = {} & Z_1 \Lambda_{11}^{-1}\{\Omega_{11} Z_1' + [\Omega_{12} - Q_1 \Pi (Q_2 \Pi)^+ \Omega_{22}] Z_2'\} x_{t-1} \\ & + Z_1 \Lambda_{11}^{-1}[Q_1 - Q_1 \Pi (Q_2 \Pi)^+ Q_2] \Psi \varepsilon_t, \end{aligned} \tag{48}$$

where $(Q_2\Pi)^+$ is the right-side pseudoinverse of $Q_2\Pi$.

When the degree of returns to scale is greater than 1.1, we have $e = s = 3$, and the system has a one-dimensional indeterminacy so there is room for sunspots to affect the forecast errors $\eta_t$ and other endogenous variables. We consider here a one-dimensional reduced-form i.i.d. sunspot shock, $\zeta_t \to \text{i.i.d.}(0, \sigma_\zeta^2)$.[10] Under indeterminacy, the forecast errors $\eta_t$ can be influenced by both the fundamental shocks $\varepsilon_t$ and the sunspot shock $\zeta_t$,

$$\eta_t = (-V_1 D_{11}^{-1} U_1' Q_2 \Psi + V_2 A)\varepsilon_t + V_2 \zeta_t, \tag{49}$$

and the full set of solutions is characterized by

$$\Gamma_0 x_t = \Gamma_1 x_{t-1} + [(I - \Pi V_1 D_{11}^{-1} U_1' Q_2)\Psi + \Pi V_2 A]\varepsilon_t + \Pi V_2 \zeta_t, \tag{50}$$

where $A$ is an arbitrary $1 \times 2$ matrix. Decompose $Z$ into $Z = [\underset{16\times 13}{Z_1} \ \underset{16\times 3}{Z_2}]$, the full set of solutions under indeterminacy is given by

$$\begin{aligned} x_t = {} & Z_1 \Lambda_{11}^{-1}[\Omega_{11} Z_1' + (\Omega_{12} - \Lambda_{12}\Omega_{22}) Z_2'] x_{t-1} \\ & + Z_1 \Lambda_{11}^{-1}(Q_1 - \Lambda_{12} Q_2)[(I - \Pi V_1 D_{11}^{-1} U_1' Q_2)\Psi + \Pi V_2 A]\varepsilon_t \\ & + Z_1 \Lambda_{11}^{-1}(Q_1 - \Lambda_{12} Q_2)\Pi V_2 \zeta_t. \end{aligned} \tag{51}$$

To put into perspective our model's dynamics in the light of those obtained by Benhabib and Wen [15] and Wen [59], we invoke the same values of parameters as in these papers when solving our model numerically, except for the two parameters pertaining to the labor market frictions, the elasticity of substitution of differentiated labor skills, $\epsilon$, and the hazard rate of wage adjustment, $\theta$, which are unique to our model, and which we set to 4 and 0.75, respectively, as reported in Table 1.[11] This allows us to examine the effect of these labor market frictions on equilibrium dynamics when compared to Benhabib and Wen [15] and Wen [59]. Also, we choose the steady-state values of $\mu_t$ and $g_t$ such that $\mu/c = 0.1$ and $g/y = 0.2$ in the steady state, and we set $\rho_\mu = \rho_g = 0.9$ and $\sigma_\mu = \sigma_g = \sigma_\zeta = 1$, while innovations in fundamentals are assumed to be orthogonal to each other and to sunspots, as in Benhabib and Wen [15].

To illustrate the effect of indeterminacy on equilibrium dynamics, we present the impulse responses of the model to the fundamental shocks under determinacy, and to both the fundamental shocks and the sunspot shock under indeterminacy. Specifically, we examine two versions of our model, one with the degree of returns to scale equal to 1.1 (the case of determinacy), and the other with the degree of returns to scale equal to 1.11 (the case of indeterminacy), as in Benhabib and Wen [15] and Wen [59]. When considering the impulse responses to one type of shocks, we turn off the other type(s) of shocks by setting the corresponding variance(s) to zero. Also, when considering the impulse responses to a consumption shock, we set the steady-state government spending-to-output ratio $g/y$ to zero; likewise, when considering the impulse responses to a government spending shock, we set the steady-state ratio $\mu/c$ to zero; and, when considering the

[10] In the case of indeterminacy, multiple sunspot shocks may influence equilibrium dynamics so the effect of sunspots is not uniquely determined. The reduced-form sunspot shock that we have considered here should be regarded as a linear combination of multiple sunspot shocks with no restriction on the weight assigned to each individual component.

[11] Our results are robust to alternative values of these two parameters within their empirically plausible ranges.

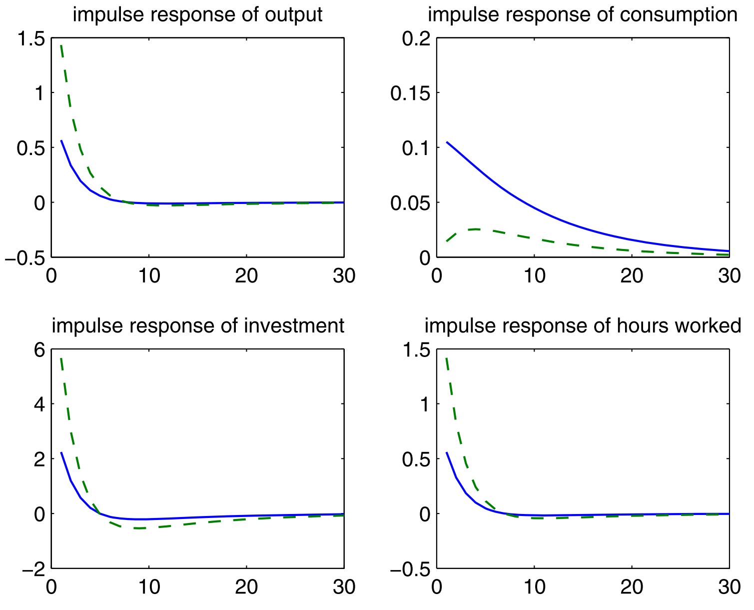


Fig. 3. Impulse responses of output, consumption, investment, and hours worked under determinacy ($\eta = 1.1$). The solid lines are responses to a consumption shock and the broken lines are responses to a government spending shock.

impulse responses to a sunspot shock under indeterminacy, we set the two steady-state ratios $g/y$ and $\mu/c$ both to zero. This is the same procedure followed by Benhabib and Wen [15] when examining their model's impulse responses to the various shocks.

Fig. 3 displays the impulse responses of output, consumption, investment, and hours worked under determinacy (e.g., $\eta = 1.1$). The solid lines are responses to a positive one-standard-deviation consumption shock and the broken lines are responses to a positive one-standard-deviation government spending shock. These responses are similar to those obtained in Benhabib and Wen [15] under determinacy: both show positive comovements among output, consumption, investment, and hours worked, with the amplitude of response in investment bigger than those in output and hours worked, which are bigger than that in consumption, as seen from the data; but, output responses in both cases are monotone, as opposed to the hump-shaped initial output responses seen in the data (e.g., Wen [59]). The magnitudes of responses are greater here than in Benhabib and Wen [15], particularly for output, investment, and hours worked, manifesting the propagation mechanism embodied in the labor market frictions.

Fig. 4 plots the impulse responses of output, consumption, investment, and hours worked under indeterminacy (e.g., $\eta = 1.11$). The solid lines are responses to a positive one-standard-deviation consumption shock, the broken lines are responses to a positive one-standard-deviation government spending shock, and the broken lines with dots are responses to a one-standard-deviation sunspot shock that leads to an upward revision in forecast errors.[12] These responses are similar to those obtained in Benhabib and Wen [15] under indeterminacy: in addition to

[12] In generating the impulse responses to fundamental shocks under indeterminacy, we set $A = 0_{1\times 2}$. Varying $A$ can vary the amplitude of the model's impulse responses to fundamental shocks under indeterminacy, corresponding to various equilibria, but not much other cyclical property of the impulse responses.

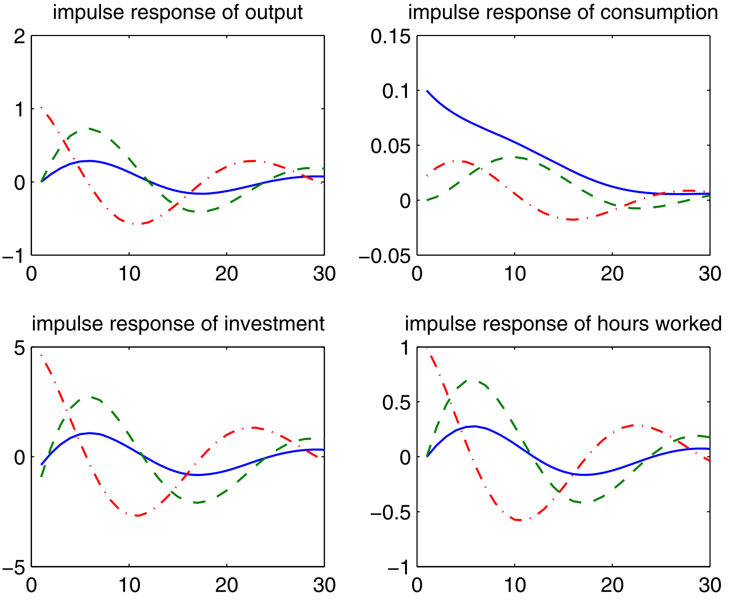


Fig. 4. Impulse responses of output, consumption, investment, and hours worked under indeterminacy ($\eta = 1.11$). The solid lines are responses to a consumption shock, the broken lines are responses to a government spending shock, and the broken lines with dots are responses to a sunspot shock.

quantity comovements and relative volatility, persistent cycles emerge following demand shocks (except for consumption's response to a consumption shock), including an i.i.d. sunspot shock, and initial output responses to serially correlated demand shocks are significantly hump-shaped. The magnitudes of responses to a sunspot shock are much greater here than in Benhabib and Wen [15], indicating that the labor market frictions hold a key for the significance of the model's response to sunspot shocks.

In sum, while the impulse responses of the model to demand shocks under indeterminacy are as reasonable as those in Benhabib and Wen [15] in terms of matching the business cycle as shown by Wen [59], sunspot shocks seem much more important here than in Benhabib and Wen [15] due to the presence of labor market frictions.

## 6. Concluding remark

We have studied the role of labor market frictions in the form of unsynchronized wage adjustment in inducing indeterminacy and sunspot-driven business cycles. Our finding is that unsynchronized wage adjustment both lowers the required degree of increasing returns for indeterminacy to an empirically plausible level and makes it invariant to the relative risk aversion in labor. In consequence, indeterminacy and sunspot-driven business cycle fluctuations can emerge for empirically plausible increasing returns regardless of the value of the relative risk aversion in labor. We have shown that indeterminacy holds the key for the propagation mechanism of the model. In particular, the model's impulse responses to demand shocks under indeterminacy are reasonable in terms of matching the business cycle, while sunspot shocks seem to become a more important source of business cycle fluctuations in the face of labor market frictions.

## Appendix A. Aggregation

In the main text, we assume that perfectly competitive retailers combine differentiated intermediate goods into a composite final good that households purchase for consumption or investment purpose, and perfectly competitive distributors combine differentiated labor services into a composite labor service that firms hire for production purpose. This assumption is made there for ease of exposition, but is not required for our results.

To see this, consider the situation in which households purchase individually differentiated goods directly from firms and firms hire individually differentiated labor services directly from households. At each date $t$, a household $i$ purchases differentiated goods $\{y_t(i,j)\}_{j\in[0,1]}$ at prices $\{P_t(j)\}_{j\in[0,1]}$ to bundle a composite basket $y_t(i)$ according to

$$y_t(i) = \left[\int_0^1 y_t(i,j)^{\frac{1}{\varepsilon}}\,dj\right]^{\varepsilon}, \tag{52}$$

which can be either consumed or invested to accumulate capital, according to (7). Minimizing the expenditure $\int_0^1 P_t(j)y_t(i,j)\,dj$ subject to (52) gives rise to household $i$'s demand for good $j$

$$y_t^d(i,j) = \left[\frac{P_t(j)}{P_t}\right]^{\frac{\varepsilon}{1-\varepsilon}} y_t(i), \tag{53}$$

where $P_t$ is a price index, as is defined in (3), and we have $P_t y_t(i) = \int_0^1 P_t(j)y_t^d(i,j)\,dj$. Summing up the demand for good $j$ from all households, $\int_0^1 y_t^d(i,j)\,di$, gives rise to the demand schedule $y_t^d(j)$ facing firm $j$, as is described by (2), where $y_t = \int_0^1 y_t(i)\,di = \int_0^1\int_0^1 P_t(j)y_t^d(i,j)\,di\,dj/P_t$, corresponding to real GDP.

At each date $t$, a firm $j$ hires differentiated labor services $\{n_t(j,i)\}_{i\in[0,1]}$ at nominal wage rates $\{W_t(i)\}_{i\in[0,1]}$ and rents capital $k_t(j)$ at real rental rate $r_t$ to produce good $j$, according to (15) where

$$n_t(j) = \left[\int_0^1 n_t(j,i)^{\frac{\epsilon-1}{\epsilon}}\,di\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{54}$$

Minimizing the production cost $\int_0^1 W_t(i)n_t(j,i)\,di + P_t r_t u_t(j)k_t(j)$ facing $j$ at $t$ for producing $y_t(j)$ subject to (15) and (54) yields firm $j$'s demand for type $i$ labor service

$$n_t^d(j,i) = \left[\frac{W_t(i)}{W_t}\right]^{-\epsilon} n_t(j), \tag{55}$$

along with (17) and (18), where $W_t$ is a nominal wage index, as is defined in (6), and we have $W_t n_t(j) = \int_0^1 W_t(i)n_t^d(j,i)\,di$. Summing up the demand for type $i$ labor service from all firms, $\int_0^1 n_t^d(j,i)\,dj$, gives rise to the demand schedule $n_t^d(i)$ facing household $i$, as is described by (5), where $n_t = \int_0^1 n_t(j)\,dj = \int_0^1\int_0^1 W_t(i)n_t^d(j,i)\,dj\,di/W_t$, corresponding to aggregate employment.

Other specifications of the model are the same while an equilibrium can be similarly defined as in the text. The rest of the analysis and equilibrium dynamics are identical to that in the text.

## Appendix B. Alternative model structure

In the main text, we assume that there are (implicit) financial arrangements that make households able to insure against idiosyncratic income risks that may arise from unsynchronized wage adjustment, so that consumption is identical across the households, although their nominal wages and hours worked may differ from one another. This assumption of consumption insurance is made in all standard models of staggered wage-setting in order to avoid an infinite-dimensional distribution in the household problem, which is clearly unworkable. See, for example, McCandless [41, Chapter 11] for a highlight on this point.

We present here an alternative model structure that does not require such financial arrangements and that generates identical equilibrium dynamics as in the main text.[13] To begin, we consider an economy in which there is a measure one of identical households. At each date $t$, a representative household purchases a composite final good $y_t$ at price $P_t$ for consumption or investment purpose, such that $y_t = c_t + \{k_{t+1} - [1 - \delta(u_t)]k_t\}$, where $c_t$ is consumption, $k_t$ is capital, $u_t \in (0, 1)$ is capacity utilization rate, and the capital depreciation rate is an increasing and convex function of the capacity utilization rate, $\delta(u_t) = u_t^{\phi}/\phi$, for $\phi > 1$. The representative household consists of a large number of family members, each endowed with a differentiated labor skill indexed by $i \in [0, 1]$. The household derives utility from its consumption of the composite final good $c_t$ and its holdings of real money balances $M_t/P_t$, while it cares about each of its members' disutility from working. Its expected lifetime utility from the viewpoint at date $t$ is

$$\mathrm{E_t} \sum_{s=t}^{\infty} \rho^{s-t} \left[ \log c_s + \psi_1 \log \frac{M_s}{P_s} - \psi_2 \int_0^1 \frac{n_s(i)^{1+\chi}}{1+\chi} \, di \right], \quad \text{for } \psi_1 > 0,\ \psi_2 > 0, \tag{56}$$

where $n_s(i)$ denotes the quantity of type $i$ labor supplied by its member $i$, and its budget constraint in period $t$ is

$$y_t + \frac{M_t}{P_t} = \int_0^1 \frac{W_t(i)}{P_t} n_t(i) \, di + r_t u_t k_t + \frac{M_{t-1}}{P_t} + f_t + \tau_t, \tag{57}$$

where $W_t(i)$ denotes the nominal wage rate for type $i$ labor, while the other notations have the same interpretations as in the main text.

Households are price takers in markets for the composite final good, capital rental, and money. Members of the households that possess differentiated labor skills are monopolistic competitors in the labor markets where they set nominal wages for their labor services in a staggered fashion à là Calvo [17], with a hazard rate $\theta$ of unable to adjusting wage which is identical and independent across time and labor type. The remaining specifications of the model are the same as in the text.

Optimal decisions on capacity utilization, consumption, and money and capital holdings by a representative household imply (12), (13), and (14), without individual household index attached. At a given date $t$, if a member $i$ of the household gets the chance to reset wage (along with members from the other households that possess the same type of labor skill), then the optimal choice of nominal wage for type $i$ labor service subject to the demand schedule (5) satisfies

[13] Such a modeling approach was originally introduced by Huang, Liu, and Phaneuf [35]. The model structure presented in this appendix is a simplified version of [35, Appendix].

$$W_t(i) = \frac{\psi_2\epsilon}{\epsilon - 1} \frac{\mathrm{E_t} \sum_{s=t}^{\infty} (\theta\rho)^{s-t} (\prod_{h=1}^{s-t} \pi_{p,t+h})^{-1} P_s n_s^d(i)^{1+\chi}}{\mathrm{E_t} \sum_{s=t}^{\infty} (\theta\rho)^{s-t} (\prod_{h=1}^{s-t} \pi_{p,t+h})^{-1} c_s^{-1} n_s^d(i)}, \tag{58}$$

with the understanding that $\prod_{h=1}^{0} \pi_{p,t+h} \equiv 1$. Substituting (5) into (58) and rearranging yield the following wage-setting rule:

$$W_t(i) = \left[ \frac{\psi_2\epsilon}{\epsilon - 1} \frac{\mathrm{E_t} \sum_{s=t}^{\infty} (\theta\rho)^{s-t} (\prod_{h=1}^{s-t} \pi_{p,t+h})^{-1} W_s^{(1+\chi)\epsilon} n_s^{1+\chi} P_s}{\mathrm{E_t} \sum_{s=t}^{\infty} (\theta\rho)^{s-t} (\prod_{h=1}^{s-t} \pi_{p,t+h})^{-1} W_s^{\epsilon} n_s c_s^{-1}} \right]^{\frac{1}{1+\chi\epsilon}}, \tag{59}$$

which is apparently identical to the wage-setting equation (11) in the main text with the individual household index to consumption removed. The rest of the analysis and equilibrium dynamics are the same as in the text.

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