ON INTEREST RATE POLICY AND EQUILIBRIUM STABILITY UNDER INCREASING RETURNS: A NOTE

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Bullard and Mitra [Journal of Monetary Economics 49 (2002), 1105–1130] find that, in a New Keynesian economy without capital and under four variants of the Taylor rule, the Taylor principle is sufficient to guarantee both determinacy and E-stability of equilibrium in most cases. Xiao [Macroeconomic Dynamics 12 (2008), 22–49] claims that with capital and mild increasing returns the Taylor principle cannot guarantee either determinacy or E-stability with any of the four rules. In this paper we show that in the Calvo-type sticky price models a second-order condition for profit maximization must be satisfied in firms’ pricing decision problem, and we point out that the examples given in Xiao’s paper to support his conclusion violate this condition. After imposing this condition, we find that increasing returns have little effect on determinacy and E-stability under two of the policy rules but significant effects under the other two. These results are obtained in models both with and without capital.

Keywords: Increasing Returns, Indeterminacy, E-stability, Taylor Principle

1. INTRODUCTION

In recent years, there has been a substantial literature in monetary economics that has studied the link between interest rate policy rules and aggregate stability. Although much of the early research was focused on the question of whether certain policy rules are associated with the possibility of indeterminacy of rational expectations equilibrium, more recently learnability (or E-stability) as an additional criterion for evaluating alternative policy rules has been emphasized. In particular, if a policy rule leads the economy to both determinacy and Stability of equilibrium, then it is viewed that the policy is properly designed and desirable [Bullard and Mitra (2002)].

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Bullard and Mitra (2002) study equilibrium determinacy and E-stability under four variants of the Taylor-type interest rate rules in a New Keynesian model. The policy rules considered by them include (i) the contemporaneous data rule, (ii) the lagged data rule, (iii) the forward expectations rule, and (iv) the contemporaneous expectations rule. Bullard and Mitra (2002) find that, in most cases, the Taylor principle with the central bank raising the nominal interest rate by more than the increase in inflation is sufficient to guarantee both determinacy and E-stability. Moreover, with rules (i) and (iv) determinacy always implies E-stability and vice versa.

The issue of determinacy and E-stability is revisited by Xiao (2008), who extends the model in Bullard and Mitra (2002) by incorporating endogenous investment and increasing returns to scale. With these extensions Xiao (2008) claims that the main conclusion in Bullard and Mitra (2002) may no longer hold, and that with very mild and empirically plausible increasing returns the Taylor principle cannot guarantee either determinacy or E-stability under any of the four policy rules.

In this paper we show that a second-order condition for optimality must be satisfied in firms’ pricing decision problem in the Calvo-type sticky price models, and we point out that the examples given in Xiao (2008) to support his conclusion violate this condition. After imposing this condition, we find that increasing returns have little effect on determinacy and E-stability under the two policy rules (i) and (iv) but significant effects under the other two.

To illustrate our point most clearly, in Section 2 we first extend Bullard and Mitra (2002) to consider a labor-only model with increasing returns; i.e., the production function for an intermediate firm is given by $N_j^\gamma$ ($\gamma > 1$). We then examine the second-order condition for profit maximization for the firm’s pricing decision problem. We find that this condition puts an upper bound on the degree of returns to scale in order to ensure the existence of an interior solution. More precisely, if $\varepsilon > 1$ denotes the negative price elasticity of individual goods in the household utility function, then the second-order condition for optimality requires that $\gamma < \varepsilon / (\varepsilon - 1)$, and this in turn implies that in the new Keynesian Phillips curve equation the coefficient for the real marginal cost (or output) must be positive, just as in Bullard and Mitra (2002). This turns out to have crucial effects on the properties of equilibrium. In fact, as also in Bullard and Mitra (2002), in most cases analytical solutions can be obtained. The results that we obtain are as follows: (a) Under the policy rules (i) and (iv) the Taylor principle still guarantees equilibrium uniqueness and E-stability with increasing returns. (b) Under the policy rules (ii) and (iii), although the qualitative results are similar to those of Bullard and Mitra (2002), quantitatively the results can be significantly affected by increasing returns, and indeed in each case the region for joint determinacy and E-stability can even approach to empty as the degree of returns to scale converges to the upper bound.

In Section 3 we show that exactly the same second-order condition for optimality applies to Xiao’s (2008) model with capital. We point out that the examples given
in Xiao (2008) to support his main results violate this condition. Again after imposing this condition, and with numerical simulations, we obtain the same results on determinacy and E-stability as in the labor-only model.

2. A LABOR-ONLY STICKY PRICE MODEL WITH INCREASING RETURNS

2.1. The Model

We modify the Calvo-type sticky price model in Bullard and Mitra (2002) by postulating a labor-only technology with increasing returns; i.e., the production function of an intermediate good of type \( j \) is given by

\[
Y_{jt} = N_{jt}^{\gamma}, \quad \gamma > 0, \tag{1}
\]

where \( \gamma \) is the degree of returns to scale in terms of the sole input, labor, and in particular when \( \gamma > 1 \) we have increasing returns. The lifetime utility of a representative household is specified as

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\chi}}{1+\chi} \right),
\]

where \( \sigma > 0, \chi \geq 0 \) and \( 0 < \beta < 1 \), \( C_t \) stands for the consumption of the final good, and \( N_t \) is the labor supply. Assume also that the final good producers ensemble the intermediate goods with a constant-returns-to-scale technology,

\[
Y_t = \left( \int_0^1 Y_{jt}^{\varepsilon-1} dj \right)^{\frac{1}{\varepsilon-1}},
\]

where \( \varepsilon > 1 \) is the elasticity of substitution of individual goods. We use \( \omega \) to denote the Calvo stickiness parameter, as in Xiao (2008).

Below we use lower case letters to denote percentage deviations of a variable from its steady state value. For the production function with increasing returns in (1), the new Keynesian Phillips curve is given by

\[
\pi_t = \beta E_t \pi_{t+1} + \frac{\kappa}{1 + A} \psi_t, \tag{2}
\]

where \( \psi_t \) represents the deviation of the real marginal cost from its steady state, \( A = \varepsilon \frac{1-\gamma}{\gamma} \) and \( \kappa = \frac{(1-\beta \omega)(1-\omega)}{\omega} \). For a derivation of equation (2), see Walsh (2005, p. 239). Under the interest rate policy feedback rules as specified in Bullard and Mitra (2002) and Xiao (2008) (see Section 2.3), the system can be reduced to two stochastic difference equations:

\[
c_t = E_t c_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1}), \tag{3}
\]

\[
\pi_t = \beta E_t \pi_{t+1} + \frac{\kappa}{1 + A} c_t. \tag{4}
\]
where \( \kappa = \frac{(1-\beta\omega)(1-\omega)}{\omega} (\sigma - 1 + \frac{\kappa + 1}{\gamma}) \). Here \( c_t, i_t, \) and \( \pi_t \) denote percentage deviations of consumption, the interest rate, and the inflation rate from their steady state values.

### 2.2. The Second-Order Condition for Optimal Pricing

In the preceding section, to obtain equations (3) and (4), we omit most details in solving the model. We now single out the firm’s pricing decision problem and, in particular, we focus on the second-order condition for optimality. We state and prove the following theorem.

**THEOREM 1.** Let \( 1 + A \equiv 1 + \frac{1 - \gamma}{\varepsilon - 1} \neq 0 \). In the labor-only model in Section 2.1 with increasing returns, the second-order necessary and sufficient condition for the existence of an interior solution for the intermediate firms’ optimal pricing decision problem is \( \gamma < \frac{\varepsilon}{(\varepsilon - 1)} \), or, equivalently, \( 1 + A > 0 \).

Proof. Note that profit maximization for the final good producers yields the demand schedule

\[
Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon} Y_t,
\]

where \( Y_t = (\int_0^1 Y_{jt} (\varepsilon - 1)/\varepsilon di)^{\varepsilon/(\varepsilon - 1)} \) is a composite good and \( P_t = (\int_0^1 P_{jt}^{1-\varepsilon} di)^{1/(1-\varepsilon)} \) is the aggregate price level.

Denote by \( W_t \) the nominal wage. From the production function in (1), the total cost for firm \( j \) at time \( t \) is given by \( TC_t(Y_{jt}) = W_t Y_{jt}^{1/\gamma} \), and the total revenue is \( TR_t(Y_{jt}) = P_{jt} Y_{jt} \). If a firm \( j \) can reset its price at time \( t \), it chooses \( P_{jt}^* \) to maximize the expected present value of its profits:

\[
\Gamma_{jt} = \sum_{s=t}^{\infty} \omega^{s-t} D_{t,s} \left[ TR_s(Y_{js}) - TC_s(Y_{js}) \right]
= \sum_{s=t}^{\infty} \xi^{s-t} D_{t,s} \left( P_{jt}^{1-\varepsilon} P_{js}^\varepsilon Y_s - P_{jt}^{-\varepsilon} \right) \left( W_s P_s^{\varepsilon} Y_s^\frac{1}{\gamma} \right),
\]

where

\[
D_{t,s} = \beta^{s-t} \left( \frac{C_s}{C_t} \right)^{-\sigma} \frac{P_t}{P_s}.
\]

The firm’s objective is to choose \( P_{jt} \) to maximize (5), taking all other variables and parameters in the expression as given. The goal is to see when the second-order condition for profit maximization is satisfied around an interior stationary point \( P_{jt}^* \).
Differentiating (5) with respect to $P_{jt}$ once and twice, we obtain

$$\frac{d\Gamma_{jt}}{dP_{jt}} = \sum_{s=t}^{\infty} \omega^{s-t} D_{t,s} \left[ -(\varepsilon - 1) P_{jt}^{-\varepsilon} P_{s}^{\varepsilon} Y_{s} + \frac{\varepsilon}{\gamma} P_{jt}^{-\frac{1}{\gamma} - 1} W_{s} P_{s}^{\frac{1}{\gamma}} Y_{s}^{\frac{1}{\gamma}} \right].$$

(6)

$$\frac{d^2 \Gamma_{jt}}{dP_{jt}^2} = \sum_{s=t}^{\infty} \omega^{s-t} D_{t,s} \left[ \varepsilon(\varepsilon - 1) P_{jt}^{-\varepsilon - 1} P_{s}^{\varepsilon} Y_{s} - \frac{\varepsilon}{\gamma} \left( \frac{\varepsilon}{\gamma} + 1 \right) P_{jt}^{-\frac{1}{\gamma} - 2} W_{s} P_{s}^{\frac{1}{\gamma}} Y_{s}^{\frac{1}{\gamma}} \right].$$

(7)

Setting (6) to zero, we see that a stationary point $P_{jt}^*$ must satisfy

$$\sum_{s=t}^{\infty} \omega^{s-t} D_{t,s} \left[ (\varepsilon - 1) P_{jt}^*-^{\varepsilon - 1} P_{s}^{\varepsilon} Y_{s} \right]$$

$$= \sum_{s=t}^{\infty} \omega^{s-t} D_{t,s} \left( \frac{\varepsilon}{\gamma} P_{jt}^*-^{\frac{1}{\gamma} - 2} W_{s} P_{s}^{\frac{1}{\gamma}} Y_{s}^{\frac{1}{\gamma}} \right) \equiv \Delta_{jt}^* > 0.$$

(8)

Evaluating (7) at the stationary point, we get

$$\left. \frac{d^2 \Gamma_{jt}}{dP_{jt}^2} \right|_{P_{jt}=P_{jt}^*} = \Delta_{jt}^* \left[ \varepsilon - \left( \frac{\varepsilon}{\gamma} + 1 \right) \right].$$

(9)

We thus have

$$\left. \frac{d^2 \Gamma_{jt}}{dP_{jt}^2} \right|_{P_{jt}=P_{jt}^*} < (>)0 \quad \text{iff} \quad \gamma < (>) \varepsilon/(\varepsilon - 1).$$

(10)

Therefore, $\gamma < \varepsilon/(\varepsilon - 1)$ is a sufficient condition for profit maximization. Because it is assumed that $1 + A \neq 0$, or equivalently, $\gamma \neq \varepsilon(\varepsilon - 1)$, the condition $\gamma < \varepsilon/(\varepsilon - 1)$ must also be a necessary condition—because if otherwise $\gamma > \varepsilon/(\varepsilon - 1)$ then the profit is minimized.4

It can be easily seen that with decreasing or constant returns, the condition $\gamma < \varepsilon/(\varepsilon - 1)$ is always satisfied. But this is not necessarily so under increasing returns. In fact, given $\varepsilon$, the second-order condition places a greater-than-unit upper bound on the degree of returns to scale to ensure the existence of an interior solution.

2.3. Equilibrium Determinacy and E-Stability

Four types of interest rate policy feedback rules are considered in Bullard and Mitra (2002), namely,

(i) Contemporaneous data rule:

$$i_t = \phi_\pi \pi_t + \phi_\gamma y_t,$$

(11)

(ii) Lagged data rule:

$$i_t = \phi_\pi \pi_{t-1} + \phi_\gamma y_{t-1},$$

(12)
(iii) Forward expectations rule:

\[ i_t = \phi_{\pi} E_t \pi_{t+1} + \phi_{y} E_t y_{t+1}, \]  

(iv) Contemporaneous expectations rule:

\[ i_t = \phi_{\pi} E_t \pi_{t-1} + \phi_{y} E_t y_{t}, \]  

where \( \phi_{\pi} \geq 0, \phi_{y} \geq 0. \)

Equilibria under the contemporaneous data rule (i) and the contemporaneous expectations rule (iv). We list the policy rules above in the same order as in Bullard and Mitra (2002). However, we now state together the results for determinacy and E-stability for rules (i) and (iv), as the results for the two cases are not only the same but also clean, as in Bullard and Mitra (2002). In particular, we have the following proposition.

**PROPOSITION 1.** Under the policy rules (i) and (iv), the necessary and sufficient condition for both determinacy and E-stability is

\[ \frac{\bar{\kappa}}{1 + A} (\phi_{\pi} - 1) + (1 - \beta) \phi_{y} > 0. \]  

We will not give proofs for this proposition and Propositions 2–3 below, as they are similar to those given in Bullard and Mitra (2002), where equation (15) is similar to equation (20) in Bullard and Mitra (2002). Most importantly, the second-order condition given in Theorem 1 implies that the term \( \frac{\bar{\kappa}}{1 + A} \) before \( (\phi_{\pi} - 1) \) in equation (15) is positive, just as in Bullard and Mitra (2002), which means that equation (15) can be rewritten as

\[ \phi_{\pi} + \frac{1 - \beta}{[\bar{\kappa} / (1 + A)]} \phi_{y} > 1. \]  

Equation (16), as elaborated in Bullard and Mitra (2002, p. 1116), corresponds to the *Taylor principle*: Nominal interest rates rise by more than the increase in the inflation rate in the long run. In particular, \( \phi_{\pi} > 1 \) is sufficient for the Taylor principle to be satisfied.

If, however, the second-order condition in Theorem 1 is violated, then \( \frac{\bar{\kappa}}{1 + A} < 0, \) and \( \phi_{\pi} + \frac{1 - \beta}{[\bar{\kappa} / (1 + A)]} \phi_{y} < 1, \) which would imply that \( \phi_{\pi} < 1 \) is sufficient for determinacy and E-stability. As a simple example, this situation could happen when \( \varepsilon / (\varepsilon - 1) = 1.05 \) and \( \gamma = 1.06 \) so that \( \gamma > \varepsilon / (\varepsilon - 1). \) In fact, as we will point out in Section 3 in the model with capital and increasing returns as in Xiao (2008), there is the same second-order condition (see Theorem 2), and all the examples in Xiao (2008) to support the main conclusion violate the second-order condition.

In Figure 1 we plot the regions of determinacy and E-stability for \( \gamma = 1, 1.02, 1.04, \) and 1.049 when \( \varepsilon / (\varepsilon - 1) = 1.05 \) and other parameter values are set at the same baseline values as in Bullard and Mitra (2002).
In Figure 1, note that corresponding to each given value of $\gamma$ there is a negatively straight line; points to the right of this line are both determinate and E-stable, whereas points to the left are both indeterminate and E-unstable. The line has a vertical intercept corresponding to $\phi_y = \bar{\kappa}/(1 + A)(1 - \beta)$ and a horizontal
intercept corresponding to $\phi_\pi = 1$, and it pivots upward as $\gamma$ increases, with the intercept staying unchanged. Thus, higher values of $\gamma$ increase only the size of the indeterminate and E-unstable region near the origin (as long as $\phi_\pi < 1$), whereas they leave the determinate and E-stable region for $\phi_\pi > 1$ unaffected.

**Equilibria under the lagged data rule (ii).** Under the lagged data rule we have the following result about equilibrium determinacy.

**Proposition 2.** Under the policy rule (ii) a set of sufficient conditions for equilibrium determinacy is

$$\frac{\bar{\kappa}}{1 + A}(\phi_\pi - 1) + (1 - \beta)\phi_y > 0, \quad (17)$$

$$\frac{\bar{\kappa}}{1 + A}(\phi_\pi - 1) + (1 + \beta)\phi_y < 2\sigma(1 + \beta). \quad (18)$$

The result in Proposition 2 is again qualitatively the same as that in Bullard and Mitra (2002). In particular, although equation (17) stands for the Taylor principle, equation (18) places upper bounds on the policy’s responses to inflation and output. Quantitatively, however, the determinacy range in the increasing-returns case here can be significantly different from that in the constant-returns case of Bullard and Mitra (2002). To see this, we fix $\varepsilon/(\varepsilon - 1) = 1.05$. By respecting the upper bound on $\gamma$ established in Theorem 1, we examine how the determinacy range varies as $\gamma$ increases. At the same time, we also examine the range for E-stability. Although no theoretical results can be obtained for E-stability, the Taylor principle seems to characterize E-stability in the lagged data policy rule, as in Bullard and Mitra (2002). Generally we find that as $\gamma$ increases, the region for joint determinacy and E-stability shrinks. The effect is most drastic as $\gamma$ converges to 1.05. This can be seen from equation (18), where the upper bound on $\phi_\pi$ tends to 1 when $\gamma$ approaches 1.05 (or, equivalently, $1 + A$ approaches 0). Indeed, the region for joint determinacy and E-stability can even become empty when $\gamma$ is sufficiently close to 1.05.

In Figure 2 we plot the regions for determinacy and E-stability for $\gamma = 1, 1.02, 1.04,$ and 1.049.

**Equilibria under the forward expectations rule (iii).** The following proposition summarizes the results under the forward expectations rule.

**Proposition 3.** Under the forward expectations rule the necessary and sufficient conditions for determinacy are

$$\frac{\bar{\kappa}}{1 + A}(\phi_\pi - 1) + (1 - \beta)\phi_y > 0, \quad (19)$$

$$\frac{\bar{\kappa}}{1 + A}(\phi_\pi - 1) + (1 + \beta)\phi_y < 2\sigma(1 + \beta). \quad (20)$$
FIGURE 2. Lagged data rule (the labor-only model).
In addition, the necessary and sufficient condition for E-stability is

$$\frac{\bar{k}}{1 + A} (\phi_\pi - 1) + (1 - \beta) \phi_y > 0. \quad (21)$$

We investigate again how the degree of returns to scale affects quantitatively the region for joint determinacy and E-stability. The results are shown in Figure 3. A similarity between the results here under the forward expectations rule and those under the lagged data rule is that an increase in $\gamma$ reduces the region of joint determinacy and E-stability, and that this region can approach to empty when $\gamma$ tends to $\varepsilon/(\varepsilon - 1)$.6

3. XIAO’S (2008) MODEL WITH CAPITAL AND INCREASING RETURNS

3.1. The Second-Order Condition for Optimal Pricing

Xiao (2008) introduces both endogenous capital and increasing returns in the Calvo-type sticky price model, and studies determinacy and E-stability under the interest rate policy rules postulated in Bullard and Mitra (2002). With the capital rental market assumption, Xiao (2008) introduces the following production function with increasing returns:

$$Y_{jt} = (K_{jt}^{\alpha} N_{jt}^{1-\alpha})^\theta, \quad 0 < \alpha < 1, \quad \theta > 0, \quad (22)$$

where $\theta$ denotes the degree of returns to scale. Among other equations of the system, we emphasize the Phillips curve equation, which is equation (29) in Xiao (2008) and takes the same form as equation (2):

$$\pi_t = \beta E_t \pi_{t+1} + \frac{\kappa}{1 + B} \phi_t, \quad (23)$$

where $B = \varepsilon \frac{1 - \theta}{\theta}$, and $\phi_t$ is the deviation of the real marginal cost from its steady state in this model with capital. Here we adopt the same notations as Xiao (2008), where, except for $\theta$, $B$, $\phi_t$, and the new parameter $\alpha$, all other notations are the same as the above labor-only model.

Importantly, just as in the labor-only model, we can establish the same result about the second-order condition for optimality in the model with capital:

**THEOREM 2.** Let $1 + B \equiv 1 + \varepsilon \frac{1 - \theta}{\theta} \neq 0$. In Xiao’s (2008) model with capital and increasing returns, the second-order necessary and sufficient condition for the existence of an interior solution for the intermediate firms’ optimal pricing decision problem is $\theta < \varepsilon/(\varepsilon - 1)$, or, equivalently, $1 + B > 0$.

Proof. See the Appendix.

The main conclusion of Xiao (2008) is that the results on determinacy and E-stability are mostly reversed in cases with small increasing returns compared to
the cases with constant returns. However, to draw this conclusion, the numerical examples given in his paper violate the second-order condition established in our Theorem 2 above. See, in particular, Table 2 in Xiao (2008), where $\theta > \varepsilon/(\varepsilon - 1)$ and $1 + B < 0$ in all of the examples.
3.2. Equilibrium Determinacy and E-stability

Here we study conditions for determinacy and E-stability with increasing returns under the interest rate policy rules in Section 2, and we do so by respecting the second-order condition stated in Theorem 2. As in Xiao (2008), we rely on
numerical simulation, because with endogenous capital analytical solutions for either determinacy or E-stability cannot be obtained. We use the same parameter values as assumed in Xiao (2008); in particular, we let $\varepsilon/(\varepsilon - 1) = 1.05$. We then examine whether and how the ranges for determinacy and/or E-stability change.

**Figure 5.** Contemporaneous expectations rule (the model with capital).
with $\theta$, and compare with the results for the case with constant returns. Our finding is that the results are largely similar to those in the above no-capital model, and they are described in Figures 4–7.\footnote{5}
From Figures 4 and 5, the degree of returns to scale has little effect on determinacy and E-stability under policy rules (i) and (iv). In particular, in each case the region of joint determinacy and E-stability changes little with $\theta$. However, under policy rules (ii) and (iii) the effects can be much more significant. Indeed,
as shown in Figures 6 and 7, in each case the region of joint determinacy and E-stability shrinks drastically as $\theta$ approaches the upper bound 1.05 in order to ensure for the existence of an interior solution.

4. CONCLUSION

In a New Keynesian economy without capital and under four variants of the Taylor rule, Bullard and Mitra (2002) find that the Taylor principle is sufficient to guarantee both determinacy and E-stability of equilibrium in most cases. Xiao (2008) claims that with capital and mild increasing returns the Taylor principle cannot guarantee either determinacy or E-stability with any of the four rules. In this paper we have established that in the Calvo-type sticky price models a second-order condition for optimality must be satisfied in firms’ pricing decision problem. This condition turns out to have crucial effects on the stability properties of equilibrium. As we have pointed out, the examples given in Xiao’s paper to support his conclusion violate this second-order condition. After imposing this condition, we have found that increasing returns have little effect on determinacy and E-stability under two of the policy rules but significant effects under the other two, in models both with and without capital.

NOTES

1. Bullard and Mitra (2002, p. 1110) assume that this sign is positive “on economic grounds.”
2. Walsh (2005) derives such an equation assuming decreasing or constant returns, i.e., $\gamma \leq 1$. It can be extended generally to include the case with increasing returns.
3. We use the notation $\kappa$ in the Phillips curve equation (2) in order to be consistent with the one used in Xiao (2008) in the model with capital (see Section 3). Then in equation (4) we use the notation $\bar{\kappa}$, which is different from what is used in Bullard and Mitra (2002) when $\gamma = 1$ or $A = 0$.
4. It should be noted that the condition $\gamma < \varepsilon / (\varepsilon - 1)$ is only a local condition for profit maximization, as the second derivative for profit is evaluated at the stationary point. Additional assumptions may be made for a global result, which we do not discuss further here. Woodford (2003, pp. 153–154) discusses global conditions for profit maximization in a similar model without increasing returns. Separately, Benhabib and Farmer (1994, p. 24) show, in a real flexible-price model with increasing returns, that the condition in Theorem 1 is sufficient for global concavity of the profit function.
5. As in Bullard and Mitra (2002), we set $\sigma = 0.157$, $\beta = 0.99$. In addition, we set $\chi = 0.12$ to fix $\bar{\kappa} = 0.024$, so when $\gamma = 1$ its value is consistent with that used in Bullard and Mitra (2002) (who use a different notation, though).
6. Notice that from equation (21) and Figure 3 in this case the range for E-stability alone is not significantly affected by increasing returns. However, the range for determinacy is greatly affected, and so is the joint region of determinacy and E-stability.
7. In our calibration in this section we follow Xiao (2008) to set $\sigma = 1$. But in the above labor-only model in Section 2, we follow Bullard and Mitra (2002) to choose $\sigma = 0.157$. We adopt this approach because under constant returns our graphs are consistent with those in the labor-only model of Bullard and Mitra (2002) or in the model with capital of Xiao (2008). The same is true in choosing the value of $\chi$ (see footnote 5). Our conclusion in this paper is not affected if we use the same set of values for $\sigma$ and $\chi$ in the two models.
REFERENCES


APPENDIX

A.1. PROOF OF THEOREM 2

The total cost for producing $Y_{jt}$ for the intermediate firm $j$ at time $t$ is given by

$$\begin{align*}
TC_t(Y_{jt}) &= \min_{K_{jt}, N_{jt}} \left( R^k_t K_{jt} + W_t N_{jt} \right),
\end{align*}$$

subject to the production function given in (22). The firm takes as given the nominal rental rate $R^k_t$ and nominal wage rate $W_t$. We can verify the following properties of the isoquant curves (in the N–K space) for (22):

$$\frac{dK}{dN} = -\frac{1 - \alpha}{\alpha} \frac{K}{N} < 0, \quad \lim_{K \to 0} \frac{dK}{dN} = 0, \quad \lim_{N \to 0} \frac{dK}{dN} = -\infty, \quad \frac{d^2K}{dN^2} = \frac{1 - \alpha}{\alpha^2} \frac{K}{N^2} > 0.$$

These properties together imply that (A.1) is a well-defined convex problem in $(K_{jt}, N_{jt})$ for any level of $Y_{jt}$ and for $\theta > 1$ with an interior solution. The first-order conditions then give rise to capital and labor demand functions as follows:

$$\begin{align*}
K^*_t(Y_{jt}) &= \left[ \frac{\alpha W_t}{(1 - \alpha) R^k_t} \right]^{1-\alpha} Y_{jt}^{\frac{1}{\theta}},
N^*_t(Y_{jt}) &= \left[ \frac{(1 - \alpha) R^k_t}{\alpha W_t} \right]^\alpha Y_{jt}^{\frac{1}{\theta}}.
\end{align*}$$

Substituting (A.2) into (A.1), we get

$$\begin{align*}
TC_t(Y_{jt}) &= \Phi \left( R^k_t, W_t; \alpha \right) Y_{jt}^{\frac{1}{\theta}},
\end{align*}$$

where

$$\Phi \left( R^k_t, W_t; \alpha \right) = R^k_t \left[ \frac{\alpha W_t}{(1 - \alpha) R^k_t} \right]^{1-\alpha} + W_t \left[ \frac{(1 - \alpha) R^k_t}{\alpha W_t} \right]^\alpha \equiv \Phi_t.$$

Using (A.3), we can write the profit function as

$$\Xi_{jt} = \sum_{s=t}^{\infty} \omega^{s-t} D_{t,s} \left( P^1_{jt} P^s Y_s - P^1_{jt} \Phi_t P^s Y_s^{\frac{1}{\theta}} \right).$$
Again, the firm’s objective is to choose $P_{jt}$ to maximize (A.4), taking all other variables and parameters in the expression as given. We now examine when the second-order condition for profit maximization is satisfied around a stationary point $P_{jt}^*$. Notice the similarity between (A.4) and (5): If we replace $\Phi$: with $W_j$ and $\gamma$ with $\theta$ in (A.4), then (A.4) becomes identical to (5). Thus following the same procedure as in Section 2.2, we have

$$\frac{d^2 \Xi_{jt}}{d P_{jt}^2} \bigg|_{P_{jt}=P_{jt}^*} < (>) 0 \quad \text{iff} \quad \theta < (>) \varepsilon / (\varepsilon - 1). \quad (A.5)$$