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The Harberger–Laursen–Metzler effect under capital market imperfections

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Abstract

This paper re-examines the effects of a permanent terms-of-trade change on a dynamic small open economy facing an imperfect world capital market as studied in Obstfeld [1982. Aggregate spending and the terms of trade: is there a Laursen–Metzler effect? Quarterly Journal of Economics 97, 251–270], which assumes that the economy faces a downward-sloping bond curve. The novelty of the present paper is that a saddle-path stable steady state comes into existence under the assumption that households' subjective discount rate is a *decreasing* function of instantaneous utility. The Harberger–Laursen–Metzler (HLM) effect is shown to hold unambiguously, i.e., an unanticipated permanent terms-of-trade deterioration leads to an increase in aggregate expenditure and a current account deficit. This result is in stark contrast to those obtained in Obstfeld [1982. Aggregate spending and the terms of trade: is there a Laursen–Metzler effect? Quarterly Journal of Economics 97, 251–270] and many other studies in the literature.

JEL classification: F41; F32

Keywords: Current account; Imperfect capital market; Terms of trade; Time preference

1. Introduction

In his seminal work, Obstfeld (1982) shows, in a framework based on intertemporal optimizing behavior of infinitely lived forward-looking agents, that an unanticipated permanent terms-of-trade deterioration leads to a reduction in aggregate expenditure, a rise in savings, and thus

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a current account surplus. This contradicts what is known as the Harberger–Laursen–Metzler effect. According to the Keynesian analysis of the effects of terms-of-trade shocks by Laursen and Metzler (1950) and Harberger (1950), an adverse shock on the terms-of-trade causes a rise in spending, a decline in saving, and a current account deficit.

Obstfeld's paper and the work by Svensson and Razin (1983)[1] have stimulated a substantial literature on examining the effects of a terms-of-trade change in dynamic optimizing frameworks.[2] While Obstfeld studies both the case with perfect capital mobility and the case with an imperfect capital market, subsequent researchers mainly focus on the case with perfect capital mobility. One reason for this, we believe, is that in the same paper Obstfeld demonstrates that his result is robust to the case with an imperfect capital market. It seems, therefore, that appending the feature of imperfect capital mobility, albeit more realistically, could only complicate the analysis without providing additional significant insights.[3]

Obstfeld uses the Uzawa (1968) utility function with the subjective discount rate being an increasing function of instantaneous utility. This assumption is crucial for his results and, under perfect capital mobility, is required for steady-state stability for an endowment-type, small open economy with a continuum of infinitely lived representative households.[4] In the extension to the case with an imperfect capital market, such an assumption is maintained in Obstfeld's paper.

The assumption of an increasing subjective discount rate, on the other hand, is a controversial one, and has been subject to criticism on intuitive grounds. It implies that the rich are more impatient. Many argue that this assumption is unappealing, and indeed Koopmans (1986) believed that the opposite case – decreasing subjective discount rate – should be a "normal case". Persson and Svensson (1985, p. 45) wrote that "the infinite horizon analysis in Obstfeld (1982) and Svensson and Razin (1983) requires that the rate of time preference increase in wealth for stability of the steady state, a restriction that is arbitrary and even counterintuitive".[5]

Using the deterministic perfect-foresight framework, the present paper revisits the issue of the effects of a permanent terms-of-trade change on the economy for the case with an imperfect capital market, where the economy faces a downward-sloping bond curve (or equivalently, an upward-sloping debt curve). This is also the case considered in Obstfeld (1982). To the contrary, the present paper shows that stable equilibrium dynamics exist under the assumption that the discount rate is a *decreasing* function of instantaneous utility (or consumption). Under the stability conditions, the Harberger–Laursen–Metzler (HLM) effect holds unambiguously, that is, an unanticipated permanent terms-of-trade deterioration leads to an increase in aggregate expenditure and a current account deficit. This result is consistent with those obtained in Laursen and Metzler (1950) and Harberger (1950) in static frameworks, and essentially

[1] Svensson and Razin (1983) start off from a two-period model and then move to an infinite-horizon model.

[2] Examples of the papers in the literature include Persson and Svensson (1985), Bean (1986), Matsuyama (1987, 1988), Sen and Turnovsky (1989), Mansoorian (1993, 1998), Servén (1999), and Ikeda (2001). See also Backus (1993), Backus et al. (1994), and Mendoza (1992, 1995), Otto (2003) for analyses using stochastic dynamic general equilibrium models. There have been mounting empirical evidences that support the existence of an HLM effect – see a survey on empirical studies in Otto (2003, Section 2).

[3] Obstfeld (pp. 263–264) wrote that "this extension adds to the complexity as well as the realism of our analysis".

[4] With a constant rate of time preference, non-degenerated, stable dynamics can also occur by introducing other sluggish state variable(s) to the system. For example, Mansoorian (1993) introduces habit persistence, while Sen and Turnovsky (1989) allow capital accumulation. However, in those models there exists dependence of steady state on initial conditions.

[5] However, we refer to Epstein (1987) and Obstfeld (1990) for supportive arguments for the assumption of increasing rate of time preference.

reverses the findings in Obstfeld (1982) in the case of imperfect capital mobility in a dynamic economy.[6]

The reason that the steady-state saddle-path stability can arise in this setup is easy to understand. That the economy faces a downward-sloping bond curve is a stabilizing factor which itself ensures a stable steady state. Decreasing rate of time preference is, on the other hand, a destabilizing factor. In the absence of the former stabilizing factor, the steady state would be unstable. Nevertheless, the stabilizing and the destabilizing forces can coexist, and as long as the stabilizing force dominates, the system still generates stable dynamics.[7]

Under the stability conditions, the long-run response of an adverse permanent terms-of-trade shock is that both expenditure and bond holdings decline. If the economy is initially at a steady state, the short-run effect requires that spending rise sharply, and then both spending and bond holdings fall gradually to the new, lower long-run levels. The long-run welfare also declines.

The rest of the paper is organized as follows. Section 2 presents the model, and the following section characterizes steady state and equilibrium dynamics. Section 4 examines the effects of a permanent deterioration in the terms of trade. Section 5 studies two extensions of the model. Section 6 offers the concluding remarks.

2. The model

Consider a small open economy populated with identical infinitely lived households. The lifetime utility of the representative household, U, is given by

$$U = \int_0^\infty u\left(h\left(c_t^{\mathrm{f}}, c_t^{\mathrm{h}}\right)\right) \mathrm{e}^{-\int_0^t \delta\left(h\left(c_v^{\mathrm{f}}, c_v^{\mathrm{h}}\right)\right)\,\mathrm{d}v}\,\mathrm{d}t, \tag{1}$$

where c_t^{f} and c_t^{h} are consumptions of the foreign and home goods at time t, $h(\cdot)$ is a positive, concave, increasing and linearly homogenous "aggregator" function, $u(\cdot)$ is the instantaneous utility function, and $\delta(\cdot)$ denotes the subjective discount rate. Note that $\delta(h(c_v^{\mathrm{f}}, c_v^{\mathrm{h}}))$ depends on the consumptions of the foreign and home goods through h.

The following assumption characterizes the instantaneous utility function $u(\cdot)$, and the discount-rate function $\delta(\cdot)$.[8]

Assumption 1.

The functions $u(\cdot)$ and $\delta(\cdot)$ are real valued, bounded above, twice continuously differentiable on $(0, \infty)$, and

(i) for any positive argument, $u(\cdot) > 0$; $u'(\cdot) > 0$; $u''(\cdot) < 0$,

(ii) for any positive argument, $\delta(\cdot) > 0$; $\delta'(\cdot) < 0$; $\delta''(\cdot) > 0$. In addition, $\delta(0) = \bar{\delta} = \sup \delta \neq \infty$, and $\inf \delta = \underline{\delta} > 0$.

[6] In this paper, we assume that bonds are denominated in terms of home good, unlike Obstfeld who assumes that bonds are denominated in terms of foreign good. If the country holds non-positive bonds (negative or zero) when an adverse terms-of-trade shock takes place, then qualitatively the results using two specifications are similar, i.e, HLM effect occurs unambiguously. However, if we used Obstfeld's specification, HLM might not arise if the country initially has positive bond holdings when the shock occurs. See the discussion in Section 4 (see also Mansoorian, 1993; pp. 157).

[7] The stability property of the present paper is similar to the closed-economy growth model in Das (2003).

[8] To avoid non-interior solutions to the household's lifetime consumption problem, we assume that $\lim_{h\to 0} u_h(h) = \infty$.

Assumption 1 follows Das (2003), except that we make an additional assumption in (ii) that $\inf \delta = \underline{\delta} > 0$, which is a sufficient condition for steady-state existence in the present open economy model. Applying the assumption to Eq. (1), the discount rate is a decreasing function of the subutility $h(\cdot)$ (and hence c_t^{f} and c_t^{h}), as well as a decreasing function of the instantaneous utility $u(\cdot)$.[9] This means that people have decreasing marginal impatience. It can be shown that Assumption 1 guarantees global monotonicity and quasi-concavity of U.

At any point of time, the household is endowed with y units of the home good and nothing of the foreign good. The home good is taken to be the numeraire, while the price of the foreign good is fixed abroad at p. The country's flow constraint at time t is

$$\dot{b}_t = r\left(\widehat{b}_t\right)b_t + y - c_t^{\mathrm{h}} - pc_t^{\mathrm{f}}, \tag{2}$$

where b_t denotes the household's net foreign asset holdings, $\widehat{b}_t$ is the aggregate net foreign asset position, both denominated in terms of the home good. $r(\widehat{b}_t)$ is the world rate of interest as a function of $\widehat{b}_t$. The number of households is normalized to unity. In solving its optimization problem, the representative household takes $\widehat{b}_t$ and hence $r(\widehat{b}_t)$ as given, but in equilibrium $\widehat{b}_t = b_t$. Assume that the world rate of interest is a positive and decreasing function of $\widehat{b}_t$, that is,

$$r\left(\widehat{b}_t\right) > 0, \quad \text{and} \quad r'\left(\widehat{b}_t\right) < 0. \tag{3}$$

Here the assumption that $r'(\widehat{b}_t) < 0$ is in line with the general notion of imperfect asset substitutability. The small open economy faces an increasing borrowing rate as its debt level increases.

We further make the following assumptions on the interest-rate function $r(\cdot)$.

Assumption 2.

(i) The interest rate elasticity with respect to asset holdings is less than unity in absolute value: $-d \ln r(b)/d \ln b < 1$.[10]

(ii) For $b \in \mathcal{S} \equiv \{b : y + r(b)b \geq 0\}$, $\sup r(b) > \bar{\delta}$, and $\inf r(b) < \underline{\delta}$, where $\bar{\delta} = \sup \delta$, and $\underline{\delta} = \inf \delta$ as defined in Assumption 1.

Part (ii) in Assumption 2 is similar to the Inada conditions for the production function in the closed economy, and is sufficient for steady-state existence. Since $r(\cdot)$ is a decreasing function, (ii) says that interest rate must be sufficiently high (greater than $\sup \delta$) for low levels of b, and sufficiently low (smaller than $\inf \delta$) for high levels of b.[11] As will be clear in the next section,

[9] Note that from Assumption 1 $\mathrm{d}\delta(h)/\mathrm{d}u(h) = (\mathrm{d}\delta/\mathrm{d}h)/(\mathrm{d}u/\mathrm{d}h) < 0$.

[10] Note that this assumption is automatically satisfied for $b \leq 0$. It is thus made effectively for $b > 0$. This assumption is also made in Obstfeld (1982).

[11] With an endogenous discount rate, usually specific conditions are needed to guarantee steady-state existence. This is true even for the case with increasing discount rate and with a fixed world interest rate, where it is required that the infimum for the discount factor be smaller than the interest rate, and that its supremum be greater than the interest rate. Although not emphasized, for steady-state existence, some other conditions are also required for the case with an imperfect capital market in Obstfeld (1982), which we do not discuss here.

part (ii) in Assumption 2 guarantees positive value of steady-state aggregate expenditure (steady-state existence is shown below, see also Eq. (15)).

The problem of the representative household is to choose a sequence of consumption levels $(c_t^{\mathrm{f}}, c_t^{\mathrm{h}})$ to maximize U, subject to the constraint (2) and the initial condition b_0.[12] Since $h(\cdot)$ is linearly homogenous, the household's problem can be solved in two stages. In the first stage, at any moment, for a given level of expenditure, z_t, the household chooses c_t^{f} and c_t^{h} to maximize $h(c_t^{\mathrm{f}}, c_t^{\mathrm{h}})$ subject to $z_t = c_t^{\mathrm{h}} + pc_t^{\mathrm{f}}$. In the second stage the household chooses the values of z_t. The first stage of the household's problem gives the indirect subutility function,

$$h(\cdot) = z_t v(p), \quad v'(\cdot) < 0. \tag{4}$$

The second stage of the household's problem is

$$\max_{z_t} \int_0^\infty u(z_t v(p)) \mathrm{e}^{-\int_0^t \delta(z_v v(p))\,\mathrm{d}v}\,\mathrm{d}t,$$

subject to

$$\dot{b}_t = r\left(\widehat{b}_t\right) b_t + y - z_t, \tag{5}$$

and the initial condition b_0. The Hamiltonian is given by

$$\mathcal{H} = u(z_t v(p)) \mathrm{e}^{-\Delta_t} + \tilde{\lambda}_t \left[r\left(\widehat{b}_t\right) b_t + y - z_t \right] - \tilde{\phi}_t \delta(z_t v(p)), \tag{6}$$

where we introduce a state variable, $\Delta_t = \int_0^t \delta(z_v v(p))\,\mathrm{d}v$. Here $-\tilde{\phi}_t$ is the co-state variable associated with the equation of motion, $\dot{\Delta}_t = \delta(z_t v(p))$. First-order conditions are

$$[u'(z_t v(p)) - \phi_t \delta'(z_t v(p))] v(p) = \lambda_t, \tag{7}$$

$$\dot{\lambda}_t = \lambda_t [\delta(z_t v(p)) - r(b_t)], \tag{8}$$

$$\dot{\phi}_t = \phi_t \delta\left(z_t v(p)\right) - u\left(z_t v(p)\right), \tag{9}$$

along with the transversality conditions. Here we use new co-state variables $\lambda_t = \tilde{\lambda}_t\,\mathrm{e}^{\Delta_t}$ and $\phi_t = \tilde{\phi}_t\,\mathrm{e}^{\Delta_t}$. It can be derived that ϕ_t is the net discounted value of lifetime utility, i.e.,

$$\phi_t = \int_t^\infty u(z_t v(p)) \mathrm{e}^{-\int_t^s \delta(z_v v(p))\,\mathrm{d}v}\,\mathrm{d}s > 0. \tag{10}$$

Log-differentiating (7) with respect to time yields the equation of motion for expenditure

$$\dot{z}_t = \frac{1}{v(p)} \left\{ \left(\frac{u' - \phi\delta'}{-u'' + \phi\delta''} \right) [r(b_t) - \delta(z_t v(p))] + \frac{\delta'\left(u - \phi\delta\right)}{\left(-u'' + \phi\delta''\right)} \right\}. \tag{11}$$

In addition, in equilibrium the flow constraint at time t is (letting $\widehat{b}_t = b_t$):

$$\dot{b}_t = r\left(b_t\right) b_t + y - z_t. \tag{12}$$

[12] We assume that the implicit constraint, that is, at each moment the present value of its output (or endowment) must exceed its indebtedness, never binds at equilibrium.

3. Steady state and equilibrium dynamics

The dynamics of the system are characterized by three differential Eqs. (9), (11) and (12). At a steady state, we have

$$\phi\delta(zv(p)) - u(zv(p)) = 0, \tag{13}$$

$$\left(\frac{u' - \phi\delta'}{-u'' + \phi\delta''}\right)[r(b) - \delta(zv(p))] + \frac{\delta'(u - \phi\delta)}{(-u'' + \phi\delta'')} = 0, \tag{14}$$

$$r(b)b + y - z = 0. \tag{15}$$

These three loci jointly determine the steady-state values for ϕ, b and z. Note that Eqs. (13) and (14) imply that

$$r(b) - \delta(zv(p)) = 0. \tag{16}$$

The steady-state values for b and z are determined by Eqs. (15) and (16). The steady state(s) for ϕ can then be obtained from Eq. (13). Combining Eqs. (15) and (16), we obtain

$$r(b) - \delta((y + r(b)b)v(p)) = 0. \tag{17}$$

Denote the LHS of Eq. (17) as $g(b)$. From part (ii) in Assumption 2, $\sup g(b) > 0$ and $\inf g(b) < 0$, so that there exists at least one value of b such that Eq. (17) holds. This can also be seen, more clearly, from a diagrammatic representation. Rewrite Eq. (17) as

$$r(b) = \delta((y + r(b)b)v(p)). \tag{18}$$

Both the LHS and RHS of Eq. (18) are downward-sloping curves when $y + r(b)b \geq 0$.[13] As $\sup r(b) > \bar{\delta}$, for sufficiently low values of $b \in \mathcal{S}$ the r-curve is above the δ-curve. However, for sufficiently high values of b, the r-curve is below the δ-curve. It is obvious that there exists at least one point of intersection, which is a steady-state value for b. This point of intersection will be unique if the two curves do not intersect again, that is, if the absolute value of the slope of the LHS remains greater than the absolute value of the slope of the RHS, or equivalently, if $r' < \delta'(r'b + r)v(p)$ holds to the right of the intersection point.

If the steady state is unique, the LHS of Eq. (18) intersects the RHS of Eq. (18) from above. If there are multiple steady states, since the r-curve must be eventually below the δ-curve, the number of steady states is odd. In particular, the steady states will be alternatively saddle-point stable and unstable, the first one and the last one being saddle points. Thus, in the case of multiple steady states, the initial conditions determine the equilibrium (i.e., depending on which

[13] We solve for the steady state(s) for b from Eq. (18) by considering only the set $b \in \mathcal{S} \equiv \{b : y + r(b)b \geq 0\}$. This will ensure, from Eq. (15), that the steady-state value(s) for the aggregate expenditure z are not negative. Moreover, all possible solutions with nonnegative steady-state values for z are included. Note that $y + r(b)b$ is increasing in b from (i) in Assumption 2. Were we to consider the larger set $b \in \mathcal{S} \cup \{b : y + r(b)b < 0\}$ when solving Eq. (18), we could possibly have more steady-state solutions for b in Eq. (18), however, corresponding to these additional solutions the implied steady-state values of the aggregate expenditure z would be negative, which must be ruled out.

steady state the economy is at or close to).[14] Fig. 1 illustrates the case with a unique steady state.

From now on we assume that there exists a unique steady state, denoted by $(\bar{b}, \bar{z}, \bar{\phi})$. To analyze the local stability, log-linearizing Eqs. (9), (11) and (12) at the steady state, we obtain

$$\begin{pmatrix} \dot{\phi} \\ \dot{z} \\ \dot{b} \end{pmatrix} = \begin{pmatrix} \delta & j_{12} & 0 \\ j_{21} & 0 & j_{23} \\ 0 & -1 & j_{33} \end{pmatrix} \begin{pmatrix} \phi \\ z \\ b \end{pmatrix}, \tag{19}$$

where $j_{12} = -v(p)\,(u' - \bar{\phi}\delta'), j_{21} = -\left[1/v(p)\right]\left[\delta'\delta/(-u'' + \bar{\phi}\delta'')\right], j_{23} = \left[1/v(p)\right]\left[(u' - \phi\delta')/(-u'' + \bar{\phi}\delta'')\right] r'(\bar{b})$, and $j_{33} = r'(\bar{b})\bar{b} + r(\bar{b}) > 0$. For the Jacobian matrix, we have

$$\text{trace} = \delta + j_{33} > 0,\ \ \det = \frac{\delta}{v(p)}\left(\frac{u' - \phi\delta'}{-u'' + \bar{\phi}\delta''}\right)\left[r' - \delta'\left(r'\bar{b} + r\right)v\left(p\right)\right].$$

Since b is the only state variable (with an initial condition), the steady state is saddle-point stable if one eigenvalue is negative, and the other two positive (or with positive real parts), which happens if $\det < 0$. Noting that $(u' - \phi\delta')/(-u'' + \bar{\phi}\delta'') > 0$ from Assumption 1 and Eq. (10), $\det < 0$ holds under the following condition,

$$r' < \delta'\left(r'\bar{b} + r\right)v\left(p\right). \tag{20}$$

However, this stability condition is satisfied at the unique steady state because the LHS of Eq. (18) intersects the RHS of Eq. (18) from above. In addition, equilibrium dynamics along the convergent path to the steady state are monotonic.

4. The Harberger–Laursen–Metzler effect

Assuming that there is a unique steady state and thus the stability condition (20) is satisfied, we first consider the effects of an increase in p on the steady-state equilibrium. Differentiating with respect to p in Eqs. (16) and (15), we have

$$r'\frac{\mathrm{d}\bar{b}}{\mathrm{d}p} - v\left(p\right)\delta'\frac{\mathrm{d}\bar{z}}{\mathrm{d}p} = \delta'\bar{z}v'\left(p\right), \tag{21}$$

$$\left(r'\bar{b} + r\right)\frac{\mathrm{d}\bar{b}}{\mathrm{d}p} - \frac{\mathrm{d}\bar{z}}{\mathrm{d}p} = 0, \tag{22}$$

from which we can solve for $\mathrm{d}\bar{b}/\mathrm{d}p$ and $\mathrm{d}\bar{z}/\mathrm{d}p$, i.e.,

$$\frac{\mathrm{d}\bar{b}}{\mathrm{d}p} = \frac{\delta'\bar{z}v'\left(p\right)}{\Gamma} < 0, \tag{23}$$

[14] The results in this and the preceding paragraphs can be shown analytically and yet more lengthily. The steady-state characterization here is similar to that in Das (2003). In contrast to her closed-economy model, however, the δ-curve in the present paper is downward-sloping instead of U-shaped. The reason is that she assumes population growth as well as the Inada conditions for the production function.

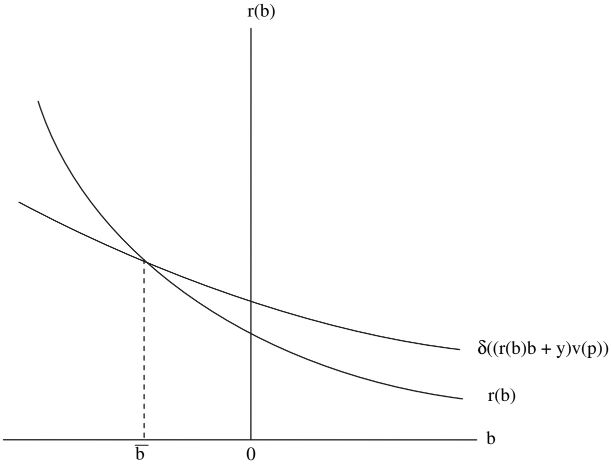

Fig. 1. Determination of the steady-state equilibrium of the net foreign asset position.

$$\frac{\mathrm{d}\bar{z}}{\mathrm{d}p} = \frac{\delta'\bar{z}v'(p)\left(r(b)b\right)'}{\Gamma} < 0, \tag{24}$$

where from Eq. (20) $\Gamma = [r' - \delta'(r'\bar{b} + r)v(p)] < 0$.

Eq. (23) says that an unanticipated permanent terms-of-trade deterioration causes a reduction in steady-state bond holdings. Since equilibrium dynamics along the transition path are monotonic, the short-run effect on the current account is qualitatively the same as the steady-state effect on bond holdings, so that the adverse permanent terms-of-trade shock unambiguously leads to a current account deficit in the short-run (and in the interim run during transition). This can also be seen from Fig. 2 where an increase in p results in an upward-shift of the δ-curve, so that steady-state bond holdings b falls. Since interest rate is a decreasing function of bond holdings, the steady-state interest rate rises. In addition, as the instantaneous discount factor $\delta(h)$ equals the interest rate in steady-state equilibrium, subutility h and hence $u(h)$ falls in steady-state equilibrium. By the monotonicity of U, this in turn implies that a terms-of-trade deterioration causes a decline in the economy's permanent, long-run utility level.

In order to see the effect on aggregate expenditure simultaneously, we can use an alternative diagrammatic representation in the z–b plane (Fig. 3). From Eq. (24) the steady-state effect on expenditure also has a definite negative sign, that is, the long-run level of expenditure must fall. To analyze the short-run effect on expenditure, notice first that steady-state values for z and b can be located on the locus $\dot{b} = 0$ projected on the two dimensional z–b plane, i.e., $r(b)b + y - z = 0$. For any specific value of p, the projection of the convergence path to the steady state for (b, z, ϕ) on the z–b plane (i.e., the FG line) is steeper than $r(b)b + y - z = 0$. To see this, if we denote the negative root of the Jacobian matrix as $\theta < 0$, and the associated eigenvector as (M, N, Q), then $\phi_t - \bar{\phi} = M\,\mathrm{e}^{\theta t}$, $z_t - \bar{z} = N\,\mathrm{e}^{\theta t}$, and $b_t - \bar{b} = Q\,\mathrm{e}^{\theta t}$. From the last row of the Jacobian matrix, we have $N + (\theta - j_{33})Q = 0$, so that $(N/Q) = -\theta + j_{33} > j_{33}$. Therefore, the optimal response to an adverse terms-of-trade shift requires a sharp increase in the aggregate spending. However, gradually aggregate spending decreases and converges along the transition path to a level lower than before the

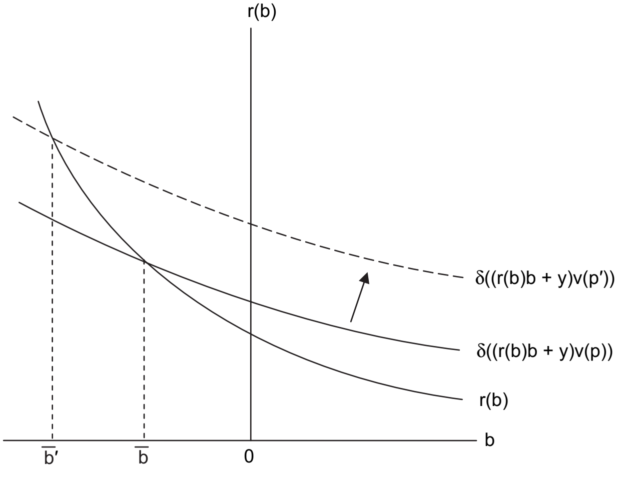

Fig. 2. Effects of a terms-of-trade deterioration on the steady-state level of the net foreign asset position.

disturbance. At the same time, the economy will keep running a deficit until bond holdings reach the new steady-state level.

We can utilize Eqs. (23) and (24) to examine other cases with an *increasing* discount rate. Let's continue to assume, as above, that bonds are denominated in terms of the home good. First, if $r' = 0$ in the case with perfect capital mobility, stability requires that $\delta' > 0$,[15] so that $\mathrm{d}\bar{b}/\mathrm{d}p > 0$, and $\mathrm{d}\bar{z}/\mathrm{d}p > 0$, which are exactly the opposite signs of Eqs. (23) and (24). A permanent terms-of-trade deterioration thus results in a current account surplus. On impact aggregate spending falls sharply, and then increases gradually to a new long-run level. In the case with an imperfect capital market and increasing rate of time preference, i.e., $r' < 0$ and $\delta' > 0$, we have $\mathrm{d}\bar{b}/\mathrm{d}p > 0$, and $\mathrm{d}\bar{z}/\mathrm{d}p > 0$. Once again, a permanent terms-of-trade worsening causes a reduction in aggregate spending and a current account surplus. These results are similar to those obtained in Obstfeld (1982) for the cases that the country does not initially have positive bond holdings (i.e., $b \leq 0$) when the terms-of-trade shock occurs. He assumes that bonds are denominated in terms of the foreign good, which implies that at any moment the household is endowed with an amount of the foreign good (i.e., rb). If the country initially has positive bond holdings, however, the signs for $\mathrm{d}\bar{b}/\mathrm{d}p$ and $\mathrm{d}\bar{z}/\mathrm{d}p$ become ambiguous. This is because a rise in p causes an increase in the value of rb when $b > 0$, so that a counter-effect arises. Similarly, ambiguity can also happen if the economy has positive endowment of the foreign good. The counter-effect may dominate if the endowment of foreign good is large enough. Obstfeld (1982) assumes zero endowment of the foreign good.

By the same logic, the unambiguous signs in Eqs. (23) and (24) come about because we have assumed that bonds are denominated in terms of the home good. These remain true qualitatively for non-positive bond holdings but may or may not hold for positive bond holdings if bonds are denominated in terms of the foreign good, depending on the relative

[15] We would need to revise the assumptions on the instantaneous utility function to ensure the monotonicity and (quasi) concavity of U. However, it does not affect the remarks here.

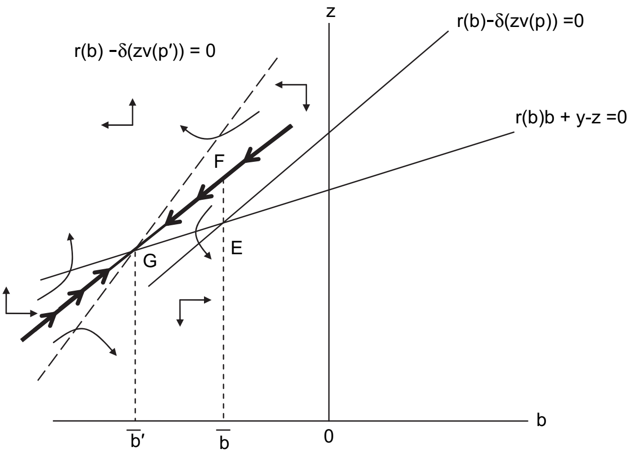

Fig. 3. The Harberger–Laursen–Metzler effect under a decreasing discount rate.

magnitudes of the opposite effects. The same thing can happen if the country has an endowment of the foreign good.

5. Extensions

In this section we study two extensions of the above model. In Section 5.1, following Schmitt-Grohé and Uribe (2003) and Neumeyer and Perri (2005), we assume that households face convex costs in adjusting bond holdings, while the world rate of interest is exogenously given. In Section 5.2, following Drugeon (1998), we consider a specification where the discount rate is a decreasing function of per-capita average consumptions, while the assumption that the economy faces a downward-sloping bond curve is maintained. In both extensions, we show that a permanent terms-of-trade change has qualitatively the same effects as the result in Section 4.

5.1. Portfolio adjustment costs

The representative household's budget constraint is,

$$\dot{b}_t = r^* b_t + y - z_t - \frac{\gamma}{2}(b_t - \bar{b})^2, \tag{25}$$

where the parameter $\gamma > 0$, and the last term on the RHS represents a quadratic cost of holding a quantity of bonds different from $\bar{b}$ (which is the steady-state bond holdings and determined below). This may be viewed as an alternative way of specifying an imperfect capital market. The assumption of a decreasing discount rate is maintained. The world interest rate, r^*, on the other hand, is exogenously given. First-order conditions again boil down to three differential equations for (ϕ_t, b_t, z_t). The form of differential equation for ϕ_t remains the same as before (Eq. (9)). However, the law of motion for aggregate expenditure is given by

$$\dot{z}_t = \frac{1}{v(p)}\left\{\left(\frac{u' - \phi\delta'}{-u'' + \phi\delta''}\right)[r^* - \gamma(b - \bar{b}) - \delta(z_t v(p))] + \frac{\delta'\left(u - \phi\delta\right)}{(-u'' + \phi\delta'')}\right\}. \tag{26}$$

Therefore, the steady state for b and z is determined by the following two loci

$$r^* - \gamma(b - \bar{b}) - \delta(zv(p)) = 0, \tag{27}$$

$$r^* b + y - z - \frac{\gamma}{2}(b - \bar{b})^2 = 0. \tag{28}$$

To ensure steady-state existence, we assume that inf $\delta < r^* <$ sup δ (see footnote 11). Solving for steady state $b = \bar{b}$ and $z = \bar{z}$ from Eqs. (27) and (28), we obtain $\bar{z} = \delta^{-1}(r^*)/v(p)$ and $\bar{b} = (\bar{z} - y)/r^*$, where $\delta^{-1}(\cdot)$ is the inverse function of $\delta(\cdot)$. Note that in contrast to the model in the previous sections, here for any given p there exists a unique steady state, which happens because the quadratic functional form for the adjustment cost is used.

It is easy to verify, and in analogous to Eq. (20), that steady-state saddle-path stability requires that

$$-\frac{\gamma}{\delta' v} > r^*. \tag{29}$$

It is clear from Eq. (29) that $\gamma > 0$ is necessary for steady-state stability, and under the stability condition the result on the terms-of-trade effects obtained in Section 4 carries over to the present model.

5.2. The discount rate depending on social consumptions

In this subsection, we consider a specification where the discount rate depends on per-capita average consumptions instead of the household's own consumptions. Specifically, assume that

$$U = \int_0^\infty u\left(h\left(c_t^{\mathrm{f}}, c_t^{\mathrm{h}}\right)\right) \mathrm{e}^{-\Omega_t}\, \mathrm{d}t, \tag{30}$$

where

$$\Omega_t = \int_0^t \delta\left(h\left(\bar{c}_v^{\mathrm{f}}, \bar{c}_v^{\mathrm{h}}\right)\right) \mathrm{d}v. \tag{31}$$

Here $\bar{c}_v^{\mathrm{f}}$ and $\bar{c}_v^{\mathrm{h}}$ denote the average levels of consumptions of the foreign and home goods, taken as given by the representative household. Everything else including the properties for the functions $\delta(\cdot)$ and $r(\cdot)$ remain the same as the model in Section 2. As pointed in Drugeon (1998), in a way related to Fisher (1987), $\delta'(\cdot) < 0$ indicates that an increase in wealth – as proxied by the social value of consumption – induces a greater valuation of future outcomes. Apart from providing an alternative specification for the discount rate, we demonstrate below that this will considerably simplify the algebra by reducing the dynamic system to two differential equations.[16] Moreover, we show below that a permanent terms-of-trade change has qualitatively the same effects as the result in Section 4.

[16] See also Shi (1999) and Schmitt-Grohé and Uribe (2003), who assume that the rate of time preference is an *increasing* function of per-capita consumptions. They all realize that analyses can be simplified under the assumption that the individual does not internalize the consumptions in the discount rate.

In solving the optimization problem, we only need to replace the last term in the Hamiltonian function in Eq. (6) in Section 2 by $\delta(\bar{z}_t v(p))$. The household takes $\bar{z}_t$ as given. The first-order condition with respect to z_t is now given by

$$u'(z_t v(p))v(p) = \lambda_t, \tag{32}$$

while other first-order conditions remain the same. Note that in contrast to the first-order condition for z_t, Eq. (7) in Section 2, in the present case in Eq. (32) the marginal utility of wealth does not depend on the net discounted value of lifetime utility ϕ_t.

The dynamic system now consists of the following three differential equations

$$\dot{\phi}_t = \phi_t \delta\big(z_t v(p)\big) - u\big(z_t v(p)\big), \tag{33}$$

$$\dot{z}_t = -\frac{1}{v(p)}\left(\frac{u'}{u''}\right)[r(b_t) - \delta(z_t v(p))], \tag{34}$$

$$\dot{b}_t = r\big(b_t\big)b_t + y - z_t. \tag{35}$$

Note that in this case the system of differential equations is recursive. In particular, equilibrium dynamics and the steady state for z and b are entirely determined by Eqs. (34) and (35). To see this, note that by setting the RHS for Eqs. (34) and (35) to zero, we obtain the same pair of equations as Eqs. (15) and (16), which are independent of ϕ. Assume that there is a unique steady state for b and z. Linearizing Eqs. (34) and (35), we obtain the sub-Jacobian matrix

$$\begin{pmatrix} \frac{1}{v(p)}\left(\frac{u'}{u''}\right)\delta' v & -\frac{1}{v(p)}\left(\frac{u'}{u''}\right) r' \\ -1 & r'\bar{b} + r \end{pmatrix},$$

whose determinant is

$$\det = \frac{1}{v(p)}\left(\frac{u'}{u''}\right)[\delta'\big(r'\bar{b} + r\big)v - r'].$$

Under the condition that $r' < \delta'(r'\bar{b} + r)v$, the steady state for b and z is saddle-point stable, so that b and z are uniquely determined along the saddle path independently of ϕ. Therefore we could even ignore ϕ_t if we are not interested in its value. This fact makes the analysis even much simpler because we can focus on the dynamics for b and z in the two by two subsystem in the b–z plane.[17] All we need to do is to replace the notations, $r(b) - \delta(zv(p))$ and $r(b) - \delta(zv(p'))$ for the two lines in Fig. 3 by the notations, $\dot{z}(p) = 0$ and $\dot{z}(p') = 0$ (see Fig. 4).[18] Then a rise of p to p' leads to a shift of the $\dot{z}(p) = 0$ line to $\dot{z}(p') = 0$ line, and if the economy is initially at point E', the economy jumps up to point F', and then

[17] In fact, we can alternatively solve the problem by writing the Hamiltonian as, $u(z_t v(p))\,\mathrm{e}^{-\Omega t} + \tilde{\lambda}_t[r(\widehat{b}_t)b_t + y - z_t]$, and ignoring ϕ_t altogether. This is so because the household takes the discount rate as given, just as in the case with a constant discount rate.

[18] The reason we use $r(b) - \delta(zv(p)) = 0$ instead of $\dot{z} = 0$ in Fig. 3 is that in that case the RHS of $\dot{z} = 0$ contains the variable ϕ (see Eq. (14)), unlike the present case in Fig. 4.

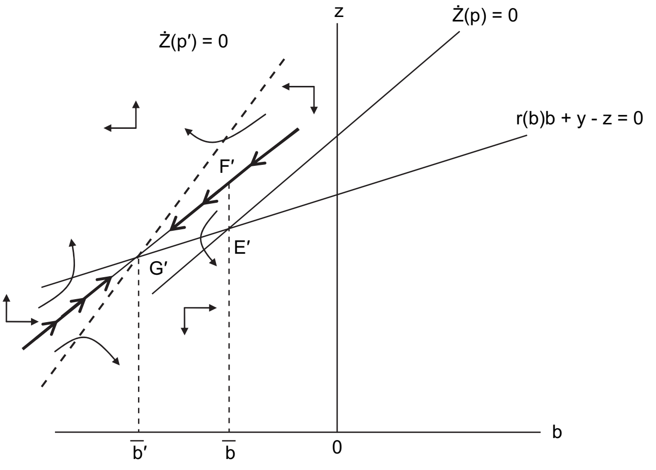

Fig. 4. The Harberger–Laursen–Metzler effect with the discount rate as a decreasing function of the per-capita average utility.

converges gradually to G' along the new stable path. Therefore, HLM effect takes place as in Section 4.

6. Conclusion

In this paper, we show that, under imperfect capital mobility, there exists a stable steady state of a small open economy with an endogenous, decreasing discount rate. This is contrary to a widely held belief. Under the stability conditions, an unanticipated permanent terms-of-trade deterioration leads to an increase in aggregate spending and a deterioration in the current account. This result is consistent with those obtained by Laursen and Metzler (1950) and Harberger (1950) in static frameworks, and essentially reverses the findings in Obstfeld (1982) in the case of imperfect capital mobility in a dynamic economy.

The framework of this paper could be used and extended in various ways, either in one-good or two-good models. First, it could be applied to study the effects of various shocks such as fiscal policy, distortionary taxation and intermediate good price shocks. Second, by incorporating endogenous labor supply and investment, one could extend the current framework to construct a small open economy RBC model with a downward-sloping bond curve (or costly portfolio adjustment) while allowing for the discount rate to be a *decreasing* function of the utility (or the average utility). Steady-state saddle-path stability would still be ensured in such a model as long as the stabilizing force from the downward-sloping bond curve (or costly portfolio adjustment) dominates the destabilizing force from the decreasing discount rate.[19] Finally, the framework could be extended to a two-country world model in a way as suggested by Devereux and Shi (1991).

[19] This stands in contrast to the benchmark small open economy RBC models considered by Schmitt-Grohé and Uribe (2003), where *either* the assumption of the discount rate being an *increasing* function of the utility (or the average utility) *or* of the economy facing a downward-sloping bond curve (or the agent facing a portfolio adjustment cost) is made to ensure steady-state stability.

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