Balanced-budget income taxes and aggregate stability in a small open economy

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Abstract

It is well known that in a closed economy real business cycle model a balanced-budget fiscal policy rule with endogenous labor (and capital) income tax rates can generate aggregate instability due to self-fulfilling expectations. This paper shows that possible worries about beliefs-driven aggregate instability associated with such a balanced-budget rule can be largely relieved for a small open economy because of the resultant income effects. We demonstrate, first analytically and then numerically, that the income effects that tend to make sunspot expectations unlikely to occur can be generated both through cross-border capital movements due to international asset trade and by endogenous terms-of-trade and other international price changes associated with international goods trade. From a policy perspective, the results of this paper suggest that if the fiscal authority relies on changes in income tax rates to achieve budget balance then integrated world capital and goods markets can play a crucial role in stabilizing the economy.

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1 Introduction

A recurrent topic of macroeconomic debate is whether a government should run a balanced-budget fiscal policy. Advocates of a balanced budget argue that persistent budget deficits can cause problems for both the economy and the government. In particular, the government’s borrowing associated with a budget deficit “crowds out” private investment spending and reduces the economy’s long run growth, and it further places financial pressure on the government’s future budgets and even its debt sustainability. Indeed, policymakers concerned about excessive deficits sometimes feel that rigid rules prohibiting – or at least setting an upper limit on – deficits are necessary. That was precisely what was stipulated in the Stability Pact in Europe in which it encouraged its members to follow the aim of balanced budgets.¹

There have been two main theoretical arguments against a balanced-budget rule. A traditional one is that it would amplify the business cycle by stimulating aggregate demand during booms via tax cuts or higher government expenditures and reducing it during recessions through fiscal contraction.² In a seminal contribution, Schmitt-Grohés and Uribe (1997, SGU henceforth) present an additional reason for why a balanced-budget rule can be destabilizing. They show that such a policy rule may induce indeterminacy and hence expectations-driven fluctuations, in a neoclassical model where the fiscal authority finances its expenditures with distortionary income taxes and labor income tax rates are endogenous.

The work of SGU (1997) has subsequently generated substantial research on balanced-budget rules and aggregate (in)stability.³ Furthermore, their results have led to a variety of policy recommendations on this issue. In particular, SGU (1997) suggest that if a balanced-budget rule is not to create endogenous aggregate instability then it should be coupled with restrictions on the fiscal authority’s ability to change tax rates. Guo and Harrison (2004) lend further support to SGU’s claim by showing that if the income tax rates are exogenous then self-fulfilling expectations cannot occur. Giannitsarou (2007) argues that because consumption taxation ensures determinacy it should be preferred to income taxation, and that a stabilization target should increase the share of public spending financed by consumption taxation.⁴

To our knowledge, existing studies on this issue in the literature including SGU (1997) have all adopted a closed economy framework. In this paper, we investigate the same issue studied in SGU (1997) in a small

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¹As part of the agreement that created the Euro, governments of member countries signed on to the Stability Pact. This agreement required each government to keep its budget deficit below 3% of the country’s GDP or it faces fines.
²As is well known, however, many economists and policymakers are also skeptical about the usefulness of counter-cyclical discretionary fiscal policy due to the time lags in its formulation and implementation.
⁴However, using a preference specification different from those used by Giannitsarou (2007), Nourry et al. (2013) find that indeterminacy can still arise under a balanced-budget consumption tax. Nishimura et al. (2013) show that indeterminacy can also occur in a two-sector model.
open economy (SOE) setting and examine whether self-fulfilling expectations and indeterminacy are relevant in such an environment. This is a realistic and necessary extension as most economies are not closed, but small and open. Indeed, under the same type of balanced-budget rule considered in SGU (1997), we find that sunspot expectations and indeterminacy are unlikely to occur in a small open economy because of the income effects associated with international asset and goods trades. This result is in stark contrast to that obtained by SGU (1997). This suggests that integrated global capital and goods markets can play a crucial role in stabilizing the economy under the balanced-budget fiscal policy by preventing persistent endogenous aggregate fluctuations from happening.

To help gain insight into our result, let’s first recall why indeterminacy can occur in SGU’s (1997) closed economy setting under the balanced-budget labor income tax rule. Consider the basic model in SGU (1997), in which public expenditures are assumed to be a constant and the fiscal authority levies a proportional tax on labor income. Taxation on labor income introduces a distortion, which affects the price of leisure in terms of the consumption goods. Suppose that households expect the tax rates to be higher. An increase in the tax rate would decrease the relative price of leisure, reducing both current consumption and labor supply through a substitution effect. However, there is also an income effect induced by taxation. If the households expect their income to be lower, they would increase their current labor supply (i.e., reduce their leisure) and curtail their consumption. Hence, the substitution and income effects influence labor supply in opposite directions. Self-fulfilling expectations can happen in SGU (1997) because the substitution effect dominates. Since the labor tax rate is counter-cyclical, a reduction in labor supply would indeed render the tax rate to be higher, making the households’ expectations self-fulfilling.

The above intuition suggests that the net effect of a tax on labor supply depends on the relative size of the substitution and income effects and hence on the specification of the utility function. SGU (1997) adopt a separable utility function that is logarithmic and has a unit intertemporal elasticity of substitution in consumption. With such a preference specification, the increase in households’ labor supply and reduction in consumption are small when expecting tax rate hikes. This is so because a large increase in labor supply and a corresponding increase in investment would require a large drop in consumption, which is impossible when the intertemporal elasticity of substitution is small. Thus, in SGU (1997), the income effect is dominated by the substitution effect. However, as is shown in this paper, if we replace in SGU (1997) by a general separable utility function with a constant intertemporal elasticity of substitution, then the relative size of the income and substitution effects can be reversed for a sufficiently large intertemporal elasticity of substitution, which would prevent the households’ expectations from becoming self-fulfilling.

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5 Even in the US, almost all the state governments follow the balanced budget rules.
6 Wen (2001) provides an intuition about indeterminacy based on households’ permanent income. The income effect we refer to here relates to the permanent income and is intertemporal. The intuition given in the present paper is also motivated by the insight provided in Giannitsarou (2007) and Anagnostopoulos and Giannitsarou (2013). See Footnotes 8 and 9 for details.
In a small open economy, in expectation of higher tax rates, the increase in households’ labor supply and investment can be large due to the income effects generated both through cross-border capital movements arising from international asset trade and by endogenous terms-of-trade and other international price changes associated with international goods trade, which give the households more flexibilities to finance the needed investment rather than simply relying on cutting consumption. In other words, the small open economy has more means to achieve consumption smoothing because of a weakened correlation between its production and consumption decisions made possible by world asset and goods market integrations. As the income effect tends to dominate the substitution effect, self-fulfilling expectations are unlikely to occur and possible worries about indeterminacy under the balanced-budget rule can be largely relieved.

We make our point in three steps. In Section 2, we present a general closed economy model that nests SGU (1997) as a special case to highlight how the intertemporal elasticity of substitution in consumption may affect the magnitude of the income effect that renders indeterminacy less possible. In particular, it is shown that, given the values of all of the other parameters, there exists an upper bound for this elasticity, above which indeterminacy cannot occur and the steady state generally exhibits saddle-path stability.

In Section 3, we present a small open economy model to demonstrate how the income effect that makes indeterminacy unlikely can be generated by cross-border capital movements due to international asset trade. To isolate such an effect of capital market integration, we assume that all international prices are exogenous and constant for the small open economy and that it can borrow and lend freely at a constant world interest rate. We show that in such a traditional SOE setting the perfectly integrated world capital market renders indeterminacy impossible, independently of the intertemporal elasticity of substitution in consumption, by bringing about a dominant income effect. When we start introducing in a somewhat ad-hoc manner impediments to international capital mobility by imposing a debt-elastic interest rate on the noncontingent international debt, we find that indeterminacy becomes possible and its likelihood gradually increases with the degree of such world capital market imperfections.

In Section 4, we present a new open economy macroeconomic model with incomplete international asset markets and multiple tradable goods, where all international prices are endogenously determined within the model, to allow for the income effects that make indeterminacy less possible to be generated not only through cross-border capital movements arising from international asset trade, but more importantly, by endogenous terms-of-trade and other international price changes associated with international goods trade, which impinge on the core mechanism of indeterminacy. In the small open economy version of such a more general and more standard open macro setting we find that, indeed, possible worries about self-fulfilling expectations and aggregate instability associated with the balanced-budget rule can be largely relieved.

Finally, Section 5 concludes the paper.
2 A closed economy model

The production side of the closed economy consists of two sectors with one producing consumption goods \((y_{1t})\) and the other producing investment goods \((y_{2t})\) according to

\[ y_{it} = k_{it}^{\alpha_i}l_{it}^{1-\alpha_i}, \quad 0 < \alpha_i < 1, \ i = 1, 2, \]  

where \(k_{it}\) and \(l_{it}\) are capital and labor inputs and \(\alpha_i\) is the share of capital in value added in sector \(i\). Factor markets are perfectly competitive, and the first-order conditions for profit maximization are

\[ w_t = (1 - \alpha_1) \frac{y_{1t}}{l_{1t}} = p_t (1 - \alpha_2) \frac{y_{2t}}{l_{2t}}, \quad r_t = \alpha_1 \frac{y_{1t}}{k_{it}} = p_t \alpha_2 \frac{y_{2t}}{k_{2t}}, \]  

where \(w_t\) and \(r_t\) are the pre-tax real wage and capital rental rates, and \(p_t\) is the relative price of investment goods to consumption goods.

A representative household maximizes the discounted present value of its lifetime utility,

\[ \int_0^\infty \left( \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \frac{l_t^{1+\chi}}{1+\chi} \right) e^{-\rho t} dt, \quad \sigma \geq 0, \ \chi > 0, \ \rho > 0, \]  

where \(c_t\) and \(l_t\) are consumption and hours worked, \(\sigma\) is the reciprocal of the intertemporal elasticity of substitution in consumption, \(\chi\) is the reciprocal of the Frisch elasticity of labor supply, and \(\rho\) is a subjective discount rate, subject to a series of budget constraint,

\[ c_t + p_t i_t + T_t = w_i l_t + r_t k_t, \]  

where \(i_t\) and \(k_t\) are investment and total capital stock, and \(T_t\) is labor income tax levied by the government that maintains a balanced budget each period,

\[ T_t = T_{1t} + T_{2t} = \tau_t w_i l_{1t} + \tau_r w_l l_{2t} = \tau_t w_i l_t = G, \]  

here, government expenditure \(G\) is pre-set and constant, \(\tau_t\) is labor tax rate, and \(T_{i_t}\) is tax on labor income earned in sector \(i\), and a series of law of motion for capital, \(\dot{k}_t = i_t - \delta k_t\). The Hamiltonian is given by

\[ \mathcal{H}_t = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \frac{l_t^{1+\chi}}{1+\chi} + \phi_t [(1 - \tau_t) w_i l_t + r_t k_t - c_t - p_t i_t] + q_t (i_t - \delta k_t). \]  

The market clearing conditions are \(l_t = l_{1t} + l_{2t}, k_t = k_{1t} + k_{2t}, c_t = y_{1t} - T_{1t},\) and \(p_t i_t = p_t y_{2t} - T_{2t}\).

Appendix A sketches the general equilibrium conditions. It is shown that, if \(\alpha_1 = \alpha_2\), the model degenerates to a one-sector setting that nests the one in SGU (1997) as a special case (i.e., \(\sigma = 1\)).

For the case with \(\alpha_1 \neq \alpha_2\), we can use (2) to obtain

\[ \frac{l_{2t}}{k_{2t}} = \nu p_t \frac{k_{2t}^{\alpha_1}}{k_{1t}^{\alpha_2}} \equiv \psi (p_t), \quad \nu \equiv \left[ \frac{\alpha_2}{\alpha_1} \left( \frac{1 - \alpha_2}{1 - \alpha_1} \right)^{-\alpha_1} \right]^{-\frac{1}{\alpha_2 - \alpha_1}}, \]  

5
and then write (5) as \( G = (1 - \alpha_2)\tau_t p_t \psi(p_t)^{-\alpha_2} \), which together imply

\[
\frac{\nu^{\alpha_2}G}{1 - \alpha_2} p_t^{\alpha_1/2} = \tau_t, \quad l_t^s = (1 - \alpha_2)\phi_t(1 - \tau_t)p_t \psi(p_t)^{-\alpha_2},
\]

where we have also used the intratemporal Euler conditions for consumption and labor in deriving the last equation. We can use the two equations in (8) to express \( l_t \) and \( \tau_t \) as functions of \( p_t \) and \( \phi_t \), and use the factor market clearing conditions to express \( k_{2t} \) as a function of \( p_t \), \( \phi_t \), and \( k_t \),

\[
k_{2t} = \frac{\alpha_1(1 - \alpha_2)}{\alpha_1 - \alpha_2} \frac{l_t(p_t, \phi_t)}{l_t(p_t)} - \frac{(1 - \alpha_1)\alpha_2}{\alpha_1 - \alpha_2} k_t.
\]

In what follows, a variable with no time index denotes the steady-state value of that variable.

The case with \( \alpha_1 = \alpha_2 (\equiv \alpha) \)

Similarly as in SGU (1997), the system reduces to two linearized dynamic equations

\[
\begin{pmatrix}
\dot{q}_t \\
\dot{k}_t
\end{pmatrix} = J \begin{pmatrix}
q_t - q \\
k_t - k
\end{pmatrix}.
\]

The trace and determinant for the Jacobian matrix are

\[
Tr(J) = \frac{\rho (\alpha + \chi)(1 - \tau) + \delta (1 - \alpha) \tau}{\chi + \alpha - (1 + \chi) \tau}, \quad Det(J) = -\frac{(\rho + \delta)^2(1 - \alpha)^2}{\alpha \sigma} \frac{\Gamma(\tau)}{\chi + \alpha - (1 + \chi) \tau},
\]

where

\[
\Gamma(\tau) = (1 + \chi)\tau^2 - \left[ \chi + (1 + \sigma + \chi) \frac{\rho + \delta(1 - \alpha)}{(\rho + \delta)(1 - \alpha)} \right] \tau + (\sigma + \chi) \frac{\rho + \delta(1 - \alpha)}{(\rho + \delta)(1 - \alpha)}.
\]

We can establish the following necessary and sufficient condition for indeterminacy.

**Proposition 1.** In the closed economy where the government follows the balanced-budget labor income tax rule (5), if \( \alpha_1 = \alpha_2 \equiv \alpha \), then the equilibrium is indeterminate if and only if

\[
\tau < \tau^0,
\]

where \( \tau \equiv \frac{\rho + \delta}{\rho + \delta(1 - \alpha)} \), and \( \tau^0 \in (0, 1) \) is the solution to the quadratic equation \( \Gamma(\tau) = 0 \).

The case with \( \sigma = 1 \) and \( \chi = 0 \) corresponds to the benchmark model in SGU (1997). While indeterminacy occurs as long as (13) holds, it can be shown that, for given values of all of the other parameters, there exists a lower bound for \( \sigma \) below which indeterminacy cannot occur.

**Corollary 1.** For given values of all of the other parameters in the model, there exists a lower bound for \( \sigma \), denoted as \( \sigma^* \), below which either the dynamic system exhibits saddle path stability or there are no equilibrium solutions, where

\[
\sigma \equiv \frac{(1 + \chi)\rho \alpha + (\rho + \delta)(1 - \alpha)^2}{\rho + \delta(1 - \alpha)} \frac{\alpha}{1 - \alpha} > 0.
\]

\( ^7 \)In (10), as \( q_t \) is a jump variable and \( k_t \) is a predetermined variable, indeterminacy happens if the two roots of \( J \) are both stable. Saddle-path stability (determinacy) occurs if only one root is stable. If the two roots are both unstable, then the system has no equilibrium solutions that converge to the steady state.
Proof. See Appendix B.

Note that as the intertemporal elasticity of substitution in consumption is the reciprocal of $\sigma$, the lower bound of $\sigma$ corresponds to the upper bound of the former, above which indeterminacy cannot occur. For the parameter values used in the benchmark model in SGU (1997), $\rho = 0.04$, $\delta = 0.1$, $\alpha = 0.33$, and $\chi = 0$, we have $\sigma = 0.3542$. Obviously, with logarithmic in consumption (i.e., $\sigma = 1$) in their paper, the lower bound condition for indeterminacy is satisfied.

The intuition for the results in Proposition 1 and Corollary 1 is as follows. Taxation on labor income introduces a distortion that affects the price of leisure in terms of the consumption goods. An increase in the tax rate decreases the relative price of leisure, reducing both current consumption and labor supply via a substitution effect. But taxation also induces an income effect. If households expect their income to be lower due to higher tax rates, they would increase current labor supply (i.e., reduce leisure) to compensate for the income loss and curtail their consumption. Thus the substitution effect and the income effect influence current labor supply in opposite directions. In Proposition 1, as in SGU (1997), self-fulfilling expectations can occur because the substitution effect dominates. Since the labor tax rate is counter-cyclical, a reduction in labor supply would indeed increase the tax rate, so that the household’s expectations are self-fulfilling.\footnote{SGU (1997) illustrate their indeterminacy result by referring to a Laffer curve, in which the tax revenue is drawn as a function of the steady-state tax rate and $\tau^o$ corresponds to the peak of the curve, under their specification of the utility function with logarithmic consumption ($\sigma = 1$) and indivisible labor ($\chi = 0$). Anagnostopoulos and Giannitsarou (2013) provide an insight into why there may exist such kind of a general connection between the shape and characteristics of the Laffer curve and indeterminacy under the balanced-budget rule and dependence of labor supply on tax rate, and this explains the condition in (13), which implies that, for indeterminacy to occur there must be a nonempty set for tax rates on the upward-sloping side of the Laffer curve in ($\tau^*, \tau^a$). We are able to verify their insight under our more general specification of the utility function by analytically showing that the upper bound $\tau^a$ on tax rate for indeterminacy indeed coincides with the tax rate corresponding to the peak of the Laffer curve for general $\sigma$ and $\chi$ (the analytical proof of this result is not presented here due to the space constraint but available upon request from the authors). This also explains, since the lower bound $\tau$ on tax rate for indeterminacy is independent of $\sigma$ but the peak of the Laffer curve shifts to the left as $\sigma$ decreases, why with large intertemporal elasticity of substitution in consumption (i.e., when $\sigma$ is small) the economy might be located on the right side of the Laffer curve even for a moderate tax rate so the government cannot collect enough revenues by raising tax rate and self-fulfilling expectations cannot occur (i.e., Corollary 1 and Appendix B).}

But this also suggests that the effect of a tax on labor income depends on the relative size of the substitution and income effects. If the intertemporal elasticity of substitution in consumption is relatively small, as in SGU (1997), in expectation of tax rate hikes, the households are only willing to cut their consumption by a small amount. A large increase in labor supply is impossible as the corresponding increase in the capital goods would require a large drop in consumption. Thus, the income effect is dominated by the substitution effect. However, the relative size of the income and substitution effects can be reversed for a sufficiently large intertemporal elasticity of substitution (i.e., a sufficiently small $\sigma$). In such a case, self-fulfilling expectations cannot happen.
The case with $\alpha_1 \neq \alpha_2$

When $\alpha_1 \neq \alpha_2$, analytical results cannot generally be obtained. We solve the model numerically. The result on indeterminacy is similar to that seen in the case with $\alpha_1 = \alpha_2$. In Figure 1, we illustrate the result by setting the same parameter values as in SGU (1997) except that we allow $\alpha_2$ to vary (it also includes the case with $\alpha_1 = \alpha_2$).\(^9\)

Figure 1 about here

In addition, for given values of all of the other parameters, it can be numerically shown that, just as in the case with $\alpha_1 = \alpha_2$, there exists a lower bound for $\sigma$ below which indeterminacy cannot occur and either the system exhibits saddle-path stability or there exist no equilibrium solutions that converge to the steady state. Moreover, if $\sigma = 0$, i.e., if the intertemporal elasticity of substitution is infinite so households are indifferent between consuming now and later, it can be analytically shown that determinacy always happens: in this case the linearized system becomes

$$
\begin{pmatrix}
\dot{q}_1 \\
\dot{k}_1
\end{pmatrix} = \begin{pmatrix}
\frac{(1-\alpha_2)(\rho+\delta)}{\alpha_1-\alpha_2} & 0 \\
-\frac{\alpha_1 \alpha_2 \delta}{\alpha_1-\alpha_2} \left( \frac{1-\alpha_1}{\alpha_1} \frac{k_1}{k_2} + \frac{1-\alpha_2}{\alpha_2} \right)
\end{pmatrix}
\begin{pmatrix}
q_1 - q \\
k_1 - k
\end{pmatrix},
$$

where the two eigenvalues $\frac{(1-\alpha_2)(\rho+\delta)}{\alpha_1-\alpha_2}$ and $-\frac{\alpha_1 \alpha_2 \delta}{\alpha_1-\alpha_2} \left( \frac{1-\alpha_1}{\alpha_1} \frac{k_1}{k_2} + \frac{1-\alpha_2}{\alpha_2} \right)$ have opposite signs, so the equilibrium is always determinate.

The basic intuition for why indeterminacy cannot occur for sufficiently small $\sigma$ remains the same as for the case with $\alpha_1 = \alpha_2$: households are willing to increase their labor supply by large magnitudes in expectation of tax hikes so that the income effect dominates. In the next section, we show that the dynamics of a traditional small open economy model with perfect capital mobility resembles that of the closed economy model here with $\sigma = 0$.

3 A traditional small open economy model

We now assume that the economy is small and open. To facilitate comparison between the results in this section and those in the previous section, we retain the two-sector economic structure and assume that the consumption goods can be traded and the investment goods cannot be traded.\(^10\)

\(^9\)The insight given in Anagnostopoulos and Giannitsarou (2013) about a general connection between the shape and characteristics of a Laffer curve and indeterminacy is also verified for this general case. Our numerical exercise shows that the upper bound $\tau^*$ on tax rate for indeterminacy coincides with the tax rate corresponding to the peak of the Laffer curve even if $\alpha_1 \neq \alpha_2$. Details of the numerical exercise are not presented here in order to conserve space but available upon request from the authors.

\(^10\)The analysis in this section focuses on the case with $\alpha_1 \neq \alpha_2$ in order to conserve space. Similar results can be obtained more easily for the case with $\alpha_1 = \alpha_2$, which can be recast as a one-sector model.
As is well known in the literature on small open economy real business cycle models, the system would have a zero eigenvalue and hence be nonstationary if, as in the closed economy model, the discount rate were assumed to be a constant.\footnote{It is also known as the knife-edge property of the steady state. The zero root problem in continuous time models corresponds to the unit root problem in discrete time setups, which are used in the small open economy literature.} Various approaches have been introduced in the literature to resolve this problem. To help simplify exposition, we here follow Bian and Meng (2004), who use a variant of a method proposed in SGU (2003), by assuming that the discount rate is an increasing function of the economywide average level of consumption $\bar{c}_t$, which is taken as given by individual households.\footnote{Making a discount rate dependent on the average levels of some variables instead of individual variables can considerably simplify the analysis. We could have directly followed SGU (2003) to make the discount rate a function of the average level of not only consumption but also hours worked, and our basic conclusion would not change. See also Shi (1999) and Meng (2006) for other examples using similar approaches.} This entails replacing $e^{-\rho t}$ in (3) with $e^{-\int_0^t \rho(s) \, ds}$, where $\rho(\cdot) > 0$.

At date $t$, domestic households of the small open economy can borrow or lend in international capital markets at a real interest rate $\theta_t$ that they take as given. For comprehensiveness, we consider here the general case with both labor and capital income taxes so the government’s balanced-budget rule takes the following generalized form, where $\zeta_t$ denotes capital tax rate,

$$
T_t = \tau_t w_t l_t + \zeta_t r_t k_t = G, \quad (16)
$$

which nests (5) as a special case (i.e., $\zeta_t = 0$). The household’s budget constraint becomes

$$
\dot{d}_t = \theta_t d_t - (1 - \tau_t) w_t l_t - (1 - \zeta_t) r_t k_t + c_t + p_t i_t, \quad (17)
$$

where $d_t$ is the household’s net international debt position. The Hamiltonian in (6) is modified accordingly. Other features of the model remain the same as in Section 2.

The model’s equilibrium system can be reduced to the following four dynamic equations

$$
\dot{p}_t = p_t \left\{ \theta_t + \delta - \alpha_2 \left[ 1 - \zeta (p_t, \phi_t) \right] \psi (p_t)^{1-\alpha_2} \right\}, \quad (18)
$$

$$
\dot{\phi}_t = \phi_t \left[ \rho \left( \phi_t^{-1/\sigma} \right) - \theta_t \right], \quad (19)
$$

$$
\dot{k}_t = [1 - (1 - \alpha_2) \tau (p_t, \phi_t) - \alpha_2 \zeta (p_t, \phi_t)] \psi (p_t)^{1-\alpha_2} k_2 (p_t, \phi_t, k_t) - \delta k_t, \quad (20)
$$

$$
\dot{d}_t = \theta_t d_t - [1 - (1 - \alpha_1) \tau (p_t, \phi_t) - \alpha_1 \zeta (p_t, \phi_t)] k_1 (p_t, \phi_t, k_t) \alpha_1 l_1 (p_t, \phi_t, k_t)^{1-\alpha_1} + \phi_t^{\frac{1}{\sigma}}, \quad (21)
$$

where we have used the first-order conditions for profit maximization, and for utility maximization with respect to consumption and labor, the factor market clearing conditions, and the government budget constraint with the endogenous labor and capital income tax rates varying in the same proportional, in deriving those implicit functions in the above system, and where we have also used the fact that $\bar{c}_t = c_t$ in equilibrium.
3.1 With perfect international capital mobility

We first consider the case where international capital mobility is perfect in the sense that domestic households can borrow and lend freely in international capital markets at an exogenously given and constant real interest rate $\theta_t = \theta$. We show below that in this case indeterminacy can never occur.

We first prove our determinacy result analytically for the case that the government follows the balanced-budget labor income tax rule (5). The linearized system in this case is given by

$$
\begin{pmatrix}
\dot{p}_t \\
\dot{\phi}_t \\
\dot{k}_t \\
\dot{d}_t
\end{pmatrix} = \begin{pmatrix}
j_{11} & 0 & 0 & 0 \\
0 & j_{22} & 0 & 0 \\
* & * & j_{33} & 0 \\
* & * & * & j_{44}
\end{pmatrix} \begin{pmatrix}
p_t - p \\
\phi_t - \phi \\
k_t - k \\
d_t - d
\end{pmatrix}.
$$

(22)

Thus the Jacobian matrix has a lower triangular form, and its four eigenvalues are

$$
\begin{align*}
j_{11} &= \frac{1 - \alpha_2}{\alpha_1 - \alpha_2} (\theta + \delta), \\
j_{22} &= -\frac{cp'(c)}{\sigma}, \\
j_{33} &= -\frac{1 - \alpha_1}{\alpha_1 - \alpha_2} \left\{ \theta [1 - (1 - \alpha_2)\tau] + \delta(1 - \alpha_2)(1 - \tau) + \frac{\delta_1(1 - \alpha_2)}{1 - \alpha_1} \right\}, \\
j_{44} &= \theta.
\end{align*}
$$

Since $j_{22} < 0$, $j_{44} > 0$, $j_{11}$ and $j_{33}$ have opposite signs, and there are two jump variables and two predetermined variables, the system is always determinate.

**Proposition 2.** In the traditional small open economy model with perfect international capital mobility, if the government follows the balanced-budget labor income tax rule (5), then the system exhibits saddle-path stability.

We now show our determinacy result for the case that the government follows the more general balanced-budget rule (16). When both labor and capital income tax rates are endogenous and vary in the same proportional and labor is indivisible, SGU (1997) find numerically that indeterminacy still arises in their closed-economy model. In the traditional small open economy model with perfect capital mobility and with indivisible labor, we can prove analytically that the system is determinate. The proof entails showing that, in this case, the Jacobian matrix for the $4 \times 4$ linearized system in the two jump and two predetermined variables, $p_t$, $\phi_t$, $k_t$, and $d_t$, again takes a lower triangular form and, of the four eigenvalues, $j_{22} < 0$ and $j_{44} > 0$, and

$$
\begin{align*}
j_{11} &= -\frac{(\theta + \delta) [\alpha_1(1 - \tau) + (1 - \alpha_2)(1 - \zeta)]}{(\alpha_2 - \alpha_1)(1 - \zeta)}, \\
j_{33} &= \frac{\left[ (1 - \alpha_1)\tau + \alpha_1\zeta \frac{k_1}{k_2} + \frac{(1 - \alpha_2)\tau + \alpha_2\zeta}{\alpha_2} \right] \delta_1\alpha_2}{(\alpha_2 - \alpha_1)\tau},
\end{align*}
$$
which always have opposite signs. This shows that the system exhibits saddle-path stability.

**Proposition 3.** In the traditional small open economy model with perfect international capital mobility, and with indivisible labor (i.e., $\chi = 0$), where the government follows the balanced-budget rule (16), the system exhibits saddle-path stability.

For the case that $\chi \neq 0$ and the government follows the balanced-budget rule (16), transparent analytical expressions for eigenvalues of the linearized system are hard to obtain. We have thus appealed to numerical exercises in confirming that indeterminacy cannot occur even in this more general case.

We have conducted other sensitivity analyses to investigate how robust our determinacy result is. For instance, in their closed economy model, SGU (1997) find that under the balanced budget rules indeterminacy also arises in cases when the labor income tax rate is endogenous and the capital income tax rate is exogenous or when both tax rates are endogenous and government purchases are income-elastic. They further extend their indeterminacy result to the case with public debt. For all of these cases, we find that indeterminacy cannot happen in the traditional small open economy model with perfect international capital mobility.

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The take-home message from the analyses in this section is thus that indeterminacy cannot occur in the traditional small open economy model with perfect international capital mobility. We provide here some intuition behind this fairly strong result. As noted in Section 2, in the closed economy, indeterminacy cannot occur when the intertemporal elasticity of substitution is large, as the income effect dominates the substitution effect. In the traditional small open economy with perfect international capital mobility, self-fulfilling expectations cannot happen simply because the income effect dominates the substitution effect, independently of the intertemporal elasticity of substitution. In expectation of higher tax rates, the increase in the households’ labor supply and the needed investment in capital goods can be large, as the households can always borrow from the outside world at a given and constant real interest rate to finance the new investment. As this effect dominates the substitution effect, labor supply increases, and the tax rates turn out to be lower, destroying self-fulfilling expectations.

It is worth highlighting the contrast between the result here and those of Weder (2001), Lahiri (2001), and Meng and Velasco (2003, 2004).\footnote{For other examples of open economy models with productive externalities and indeterminacy, see Nishimura and Shimomura (2002a, 2002b) and Hu and Mino (2013).} In traditional small open economy models with external effects in production, those authors all find that indeterminacy is more likely to arise with a perfect world capital market than otherwise in the closed economy. In fact, indeterminacy happens in their setups because of the

\footnote{These additional results are available upon request from the authors.}
strong external effects on the rental returns to physical capital. Expectations of higher future returns to capital call for an increase in current investment. In a closed economy with these kinds of external effects, a larger intertemporal elasticity of substitution in consumption actually makes it easier for indeterminacy to occur, because if the households want to invest more they are more willing to adjust their consumption. In the presence of strong external effects in technology indeterminacy can happen more easily in those traditional small open economy models because the households who want to invest more can borrow abroad without reducing their consumption level, independently of the intertemporal elasticity of substitution in consumption.

### 3.2 With imperfect international capital mobility

In Section 3.1, the traditional small open economy is assumed to have perfect access to world capital markets in the sense that it can borrow from or lend to the rest of the world at a given and constant real interest rate. In such a case indeterminacy and self-fulfilling expectations can be ruled out. On the other hand, the case of a closed economy examined in Section 2 and SGU (1997) in which indeterminacy can arise could be recast as an open economy under financial autarky where borrowing from or lending to the rest of the world is never allowed or forbiddingly costly. In this section, we consider an intermediate case between the two extremes, where domestic households face an imperfect world capital market, to investigate the sensitivity of the results in Section 3.1 to varying degrees of international capital market integration.\(^\text{15}\)

Following some of the literature on traditional small open economy models (e.g., SGU 2003), we model world capital market imperfections by assuming that the interest rate faced by domestic households, \(\theta_t\), is increasing in the average level of the country’s outstanding debt, denoted by \(\bar{d}_t\) and taken as given by the households.\(^\text{16}\) Specifically, \(\theta_t\) is here given by

\[
\theta_t = \theta + \omega(\bar{d}_t), \quad \omega'(\cdot) > 0, \tag{23}
\]

where \(\theta\) is a fixed world interest rate as in Section 3.1, and \(\omega(\cdot)\) is a country-specific interest-rate risk premium that is meant to capture world capital market imperfections.

Transparent analytical demonstrations are hard to show for this case, so in what follows we focus on showing our results numerically. To do so, we need to assign values to the model’s parameters. We set \(\rho(c) = 0.04\), \(\theta = 0.04\), \(\delta = 0.1\), \(\sigma = 1\), \(\chi = 0\), and \(d = 0.7442\), as in SGU (1997, 2003), and we choose \(\alpha_1 = 0.33\) and \(\alpha_2 = 0.3\). Next we parameterize \(\omega(\cdot)\) following SGU (2003):

\[
\omega(\bar{d}_t) = \gamma(e^{\bar{d}_t - d} - 1), \tag{24}
\]

\(^{15}\)We are grateful to two anonymous referees for suggesting that we examine such an intermediate case.

\(^{16}\)Similar results can be obtained if we instead assume that the households face a cost of holding or issuing bonds.
where $\gamma > 0$ governs the elasticity of the interest rate faced by the country with respect to changes in the country’s indebtedness, thus a greater value of $\gamma$ implies a greater degree of world capital market imperfections. This is a key parameter for our analysis in this section. We consider 0.00074 as a benchmark value for $\gamma$ following the calibration of SGU (2003) based on Canadian data, but at the same time we examine how varying the value of $\gamma$ may affect our determinacy result.

We find that indeterminacy indeed becomes possible in the presence of world capital market imperfections and its likelihood increases with the degree of such imperfections. Figure 2 illustrates this for the case when the government follows the balanced-budget labor income tax rule (5). The figure shows that indeterminacy becomes possible when $\gamma$ becomes positive, and the range of the steady-state labor income tax rate for which indeterminacy occurs widens with $\gamma$.

Figure 2 about here

Figure 3 further demonstrates this for the case that the government follows the more general balanced-budget rule (16). As the figure shows, the configurations of the steady-state labor and capital income tax rates under which indeterminacy occurs constitute a nonempty set if $\gamma$ is positive, and the set becomes larger as $\gamma$ gets bigger. To get some empirical feel, the figure also displays the pairs of labor and capital income tax rates estimated by Mendoza et al. (1997) for eleven developed small open economies, including Austria (AT), Australia (AU), Canada (CA), Denmark (DK), Finland (FI), Netherlands (NL), Norway (NO), New Zealand (NZ), Spain (SP), Sweden (SE), and Switzerland (SZ). As can be seen from the figure, for the benchmark value of $\gamma$, there are no countries for which the tax rate pairs fall into the indeterminacy region. For larger $\gamma$, some countries may fall into the indeterminacy region. For example, were $\gamma$ four times bigger than its benchmark value, the indeterminacy region would expand to include three of the eleven countries.

Figure 3 about here

Given the results for the two extreme cases, the results in this section are intuitive to explain. In the case with perfect international capital mobility, a large income effect is possible because, when households expect higher tax rates, they can sufficiently increase their labor supply as they can always borrow from abroad at a fixed interest rate to finance the needed investment goods, preventing the occurrence of self-fulfilling expectations. If the households face a steeply upward-sloping debt supply curve, however, as adding more investment goods by borrowing from overseas can quickly become very costly, the households would increase less of their labor supply. This would weaken the income effect. Indeterminacy becomes possible if the income effect is dominated by the substitution effect, just as in the case of a closed economy.

It is also worth noting that, for indeterminacy to occur here, the intertemporal elasticity of substitution in consumption must be relatively low, or, its reciprocal $\sigma$ must be relatively large. This is so since, when facing high borrowing cost, the households who want to investment more could alternatively reduce their
consumption. However, reducing consumption can also be costly with a low intertemporal elasticity of substitution, leading to less increase in investment and labor supply. Thus, low intertemporal elasticity of substitution is another factor that weakens the income effect. In fact, for any given value of $\gamma$, one can always find a value for $\sigma$ below which indeterminacy can never happen, just as in the case of a closed economy. Hence, in contrast to the case with perfect international capital mobility in which production and consumption decisions are effectively separated, world capital market imperfections essentially re-link production and consumption decisions, which in turn can cause equilibrium indeterminacy and beliefs-driven aggregate fluctuations in the traditional small open economy model.

4 A new open economy macroeconomic model

The traditional small open economy model adopted in Section 3.1 allows us to show in a transparent way how a perfectly integrated (though incomplete) world capital market may render indeterminacy impossible by introducing a dominant income effect. Yet this transparency comes at a cost that all international prices are assumed to be exogenous and constant for the small open economy. While Section 3.2 provides a quick way to illustrate how relaxing this assumption may bring the possibility of indeterminacy back, this is done in an ad-hoc manner by imposing a debt-elastic interest rate on the noncontingent international debt. It would be interesting to conduct such exercises in a new open economy macroeconomic model with incomplete international asset markets and multiple tradable goods where all international prices (i.e., terms of trade, exchange rate, interest rate) are endogenously determined within the model, as this would allow us to explore the implications of international price adjustments that impinge on the core mechanism of indeterminacy. This is what we do in this section. We find that, in such a more general and more standard open macro model, the income effects that tend to make indeterminacy less likely to occur can be generated not only through cross-border capital movements due to international asset trade, as in the traditional small open economy model presented in Section 3, but more importantly, by endogenous terms-of-trade and other international price changes associated with international goods trade.17

To be in line with much of the literature of the new open economy macro models that use discrete-time setups, we adopt a discrete-time setting in this section. We shall proceed by first setting up a general two-country new open economy macroeconomic model (e.g., Corsetti, Dedola and Leduc 2008), and then focusing our analysis on the case where the home country is a small open economy. In what follows, a variable for the foreign country (i.e., the rest of the world) is super indexed by a star.

17We are grateful to Giancarlo Corsetti (the editor) for suggesting that we examine our results in such a more general model.
4.1 A general two-country setup

4.1.1 Firms

The world economy consists of two countries, Home and Foreign, each producing tradable intermediate goods using a Cobb-Douglas technology. Particularly, the production function for Home goods $y_t$ is given by

$$y_t = k_t^{\alpha} l_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where $k_t$ and $l_t$ are Home capital and labor inputs. Factor markets are perfectly competitive, where the first-order conditions for cost minimization imply that

$$w_t = (1 - \alpha) \left( \frac{k_t}{l_t} \right)^{\alpha} \left( \frac{y_t}{l_t} \right), \quad (26)$$

$$r_t = \alpha \left( \frac{l_t}{k_t} \right)^{1-\alpha} \frac{y_t}{k_t}, \quad (27)$$

where $w_t$ and $r_t$ are the pre-tax real wage rate and capital rental rate.

4.1.2 Final goods and price indexes

Home combines Home produced goods $y_{H,t}$ and Foreign produced goods $y_{F,t}$ to fabricate Home final goods $z_t$ according to

$$z_t = \left[ \mu^{\frac{\eta}{1-\eta}} y_{H,t}^{\frac{\eta}{1-\eta}} + (1 - \mu)^{\frac{\eta}{1-\eta}} y_{F,t}^{\frac{\eta}{1-\eta}} \right]^\frac{1}{\eta}, \quad \eta > 0, \quad (28)$$

where $\mu = 1 - (1 - n) a$ is the relative weight of Home-produced goods in the composite, which is a function of the relative size of the Home economy, $n$, and the degree of its trade openness, $a \in (0, 1)$.

Denote by $P_{H,t}$, $P_{F,t}$, and $P_t$ the prices of Home produced goods, Foreign produced goods, and Home final goods, all denominated in Home currency units. Cost minimization yields the following demand schedules of Home households

$$y_{H,t} = \mu \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} z_t, \quad y_{F,t} = (1 - \mu) \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} z_t, \quad (29)$$

and the following relation among the three prices

$$P_t = \left[ \mu P_{H,t}^{1-\eta} + (1 - \mu) P_{F,t}^{1-\eta} \right]^\frac{1}{1-\eta}. \quad (30)$$

Foreign combines Foreign produced goods $y_{F,t}^*$ and Home produced goods $y_{H,t}$ to fabricate Foreign final goods $z_t^*$ according to

$$z_t^* = \left[ \mu^*^{\frac{\eta}{1-\eta}} y_{F,t}^{\frac{\eta}{1-\eta}} + (1 - \mu^*)^{\frac{\eta}{1-\eta}} y_{H,t}^{\frac{\eta}{1-\eta}} \right]^\frac{1}{\eta}, \quad \eta > 0, \quad (31)$$

where $\mu^* = 1 - na^*$, and $a^* \in (0, 1)$ measures the degree of Foreign’s trade openness.
Denote by $P^*_F; t$, $P^*_H; t$, and $P^*_t$ the prices of Foreign produced goods, Home produced goods, and Foreign final goods, all denominated in Foreign currency units. Cost minimization yields the following demand schedules of Foreign households

$$y^*_F; t = \mu^* \left( \frac{P^*_F; t}{P^*_t} \right)^{-\eta} z^*_t, \quad y^*_H; t = (1 - \mu^*) \left( \frac{P^*_H; t}{P^*_t} \right)^{-\eta} z^*_t,$$

(32)

and the following relation among the three prices

$$P^*_t = \left[ \mu^* P^*_F; t \frac{1}{1 - \mu^*} + (1 - \mu^*) P^*_H; t \frac{1}{1 - \mu^*} \right]^{\frac{1}{1-\eta}}. \quad (33)$$

4.1.3 Exchange rate and terms of trade

Denote by $S_t$ the nominal exchange rate measured by the price of Foreign currency in units of Home currency. We assume that the law of one price holds for Home- and Foreign-produced goods so that

$$P^*_H; t = S_t P^*_H; t, \quad P^*_F; t = S_t P^*_F; t. \quad (34)$$

The real exchange rate $\varepsilon_t$ and Home terms of trade $s_t$ are

$$\varepsilon_t \equiv S_t P^*_F; t \frac{1}{P^*_t}, \quad s_t \equiv P^*_F; t \frac{1}{P^*_H; t} = S_t P^*_F; t. \quad (35)$$

which can be shown to have the following relationship

$$\left( \frac{1}{\varepsilon_t} \right)^{1-\eta} = \frac{\mu + (1 - \mu) s_t^{1-\eta}}{(1 - \mu^*) + \mu^* s_t^{1-\eta}}. \quad (36)$$

4.1.4 Households

Home economy is inhabited by a large number of identical infinitely-lived households. The representative household maximizes the expected present value of its lifetime utility

$$\max E_0 \sum_{t=0}^{\infty} \xi_t \left( \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - B_t^{1+\chi} \frac{1 + \chi}{1} \right), \quad \sigma \geq 0, \quad \chi \geq 0, \quad 0 < \beta < 1, \quad (37)$$

where $c_t$ denotes consumption of Home final goods. To avoid the non-stationarity problem of the net foreign asset position, we follow much of the literature on the new open economy macro models with incomplete international asset markets (e.g., Corsetti et al. 2008) to assume an endogenous discount factor that is a function of the average per capita level of consumption $\bar{c}_t$, and hours worked $\bar{l}_t$, which are taken as given by individual households even though $c_t = \bar{c}_t$ and $l_t = \bar{l}_t$ hold in equilibrium.

Traded on international asset market is a non-contingent bond denominated in Home currency. Denote by $B_{t+1}$ the quantity of the bond purchased by Home households at date $t$ that will be paid off at date $t + 1$
and \( D_t \) the price of the bond. Home households face the following sequence of budget constraints,

\[
P_{H,t}y_{H,t} + P_{F,t}y_{F,t} + P_{H,t}T_t = P_{H,t}(w_tI_t + r_tk_t) + B_t - D_tB_{t+1},
\]

which can be rewritten, noting that \( P_{H,t}y_{H,t} + P_{F,t}y_{F,t} = P_tz_t \), as

\[
\frac{P_t}{P_{H,t}}z_t + T_t = w_tI_t + r_tk_t + \frac{B_t - D_tB_{t+1}}{P_{H,t}},
\]

where \( T_t \) is labor income tax levied by Home government that maintains a balanced budget each period,

\[
T_t = \tau_tw_t = G,
\]

where government expenditure \( G \) is pre-set and constant, and the labor tax rate \( \tau_t \) is endogenous.

The period-\( t \) budget constraint can then be rewritten as

\[
\frac{D_tB_{t+1} - B_t}{P_{H,t}} = (1 - \tau_t)w_tI_t + r_tk_t - \frac{P_t}{P_{H,t}}z_t,
\]

where \( z_t \) can be either consumed \( (c_t) \) or invested \( (i_t) \) so that

\[
z_t = c_t + i_t,
\]

and the law of motion for capital is

\[
k_{t+1} = (1 - \delta)k_t + i_t.
\]

At each date \( t \), the household’s objective is to maximize

\[
\mathbb{E}_t \sum_{s=t}^{\infty} \frac{\xi_s}{\xi_t} \left\{ \left( \frac{c_t^{1-\sigma} - 1}{1-\sigma} - B_{t+1} \right) + \phi_s \left( 1 - \tau_s \right) w_sI_s + r_s \frac{P_{H,s}}{P_{H,t}} (c_s + i_s) - \frac{D_sB_{t+1} - B_s}{P_{H,s}} \right\}
\]

\[
+ q_s [i_s + (1 - \delta)k_s - k_{s+1}]
\]

The first-order optimization conditions for \( c_t, I_t, i_t, B_{t+1} \), and \( k_{t+1} \) are, respectively,

\[
c_t^{1-\sigma} = \phi_t \frac{P_t}{P_{H,t}},
\]

\[
Bl_t = (1 - \tau_t) \phi_t w_t,
\]

\[
q_t = \phi_t \frac{P_t}{P_{H,t}},
\]

\[
D_t \frac{\phi_t}{P_{H,t}} = \beta (\tau_t, \xi_t) \mathbb{E}_t \left( \frac{\phi_{t+1}}{P_{H,t+1}} \right),
\]

\[
q_t = \beta (\tau_t, \xi_t) \mathbb{E}_t [\phi_{t+1}r_{t+1} + (1 - \delta)q_{t+1}].
\]
4.1.5 International risk sharing

Combining the first-order optimization conditions for \( c_t \) and \( B_{t+1} \) above gives rise to

\[
D_t = \beta (\tau_t, \lambda_t) E_t \left( \frac{c_t^\sigma}{c_{t+1}^\sigma} \frac{P_t}{P_{t+1}} \right). \tag{49}
\]

Foreign households’ first-order optimization conditions for consumption and bond holdings imply

\[
D_t = \beta \left( \tau_t^*, \lambda_t^* \right) E_t \left( \frac{c_t^\sigma}{c_{t+1}^\sigma} \frac{P_t^*}{P_{t+1}^*} \frac{P_{H,t}}{P_{H,t+1}} \right),
\]

which, given the law of one price for Home-produced goods \( (P_{H,t} = S_t P_{H,t}^*) \), reduces to

\[
D_t = \beta \left( \tau_t^*, \lambda_t^* \right) E_t \left( \frac{c_t^\sigma}{c_{t+1}^\sigma} \frac{P_t}{P_{t+1}^*} \frac{S_t}{S_{t+1}} \right),
\]

which, in light of the relation \( \epsilon_t = S_t P_{H,t}^*/P_t \), further reduces to

\[
D_t = \beta \left( \tau_t^*, \lambda_t^* \right) E_t \left( \frac{c_t^\sigma}{c_{t+1}^\sigma} \frac{P_t}{P_t^*} \frac{\epsilon_t}{\epsilon_{t+1}} \right).
\]

We therefore have

\[
\beta (\tau_t, \lambda_t) E_t \left( \frac{c_t^\sigma}{c_{t+1}^\sigma} \frac{P_t}{P_{t+1}} \right) = \beta \left( \tau_t^*, \lambda_t^* \right) E_t \left( \frac{c_t^\sigma}{c_{t+1}^\sigma} \frac{P_t}{P_{t+1}^*} \frac{\epsilon_t}{\epsilon_{t+1}} \right). \tag{50}
\]

4.1.6 Market clearing condition for Home-produced goods

The market clearing condition for Home-produced goods,

\[
n (y_t - T_t) = ny_{H,t} + (1 - n) y_{H,t},
\]

can be rewritten as

\[
y_t - T_t &= y_{H,t} + \frac{1 - n}{n} y_{H,t}^*
\]
\[
&= \mu \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} z_t + \frac{1 - n}{n} (1 - \mu^*) \left( \frac{P_{H,t}}{P_t^*} \right)^{-\eta} z_t^*
\]
\[
&= [1 - (1 - n) a] \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} z_t + [(1 - n) a^*] \left( \frac{P_{H,t}}{P_t^*} \right)^{-\eta} z_t^*
\]
\[
&= \left\{ \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} \right\} [1 - (1 - n) a] z_t + [(1 - n) a^*] \left( \frac{c_t^\sigma}{c_{t+1}^\sigma} \right) \epsilon_t \epsilon_{t+1}, \tag{51}
\]

where the second equality follows from the demand schedules (29) and (32), the third equality holds because \( \mu = 1 - (1 - n) a \) and \( \mu^* = 1 - na^* \), and the last equality follow from the first relations in (34) and (35) that imply \( P_{H,t}^*/P_t^* = P_{H,t}/(\epsilon_t P_t) \).
4.2 Home as a small open economy

We shall now focus on the case where Home is a small open economy, that is, $n \to 0$, $\mu \to 1 - a$ and $\mu^* \to 1$. Then (36) reduces to

$$\varepsilon_t^{-(1-\eta)} = (1 - a) s_t^{-(1-\eta)} + a, \quad (52)$$

and the prices of Home and Foreign aggregate consumption bundles reduce to

$$P_t = \left[(1 - a) P_{H,t}^{1-\eta} + a P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \quad P_t^* = P_{F,t}^*, \quad (53)$$

the latter of which implies that

$$s_t = \frac{S_t P_t^*}{P_{H,t}}, \quad (54)$$

which can be used to establish the following relationships,

$$\frac{P_t}{P_{H,t}} = \frac{s_t}{\varepsilon_t} = \left[(1 - a) + a s_t^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv \varphi(s_t). \quad (55)$$

The market clearing condition (51) for Home-produced goods simplifies to

$$y_t - T_t = (1 - a) \varphi(s_t)^z z_t + a^* s_t^y z_t^*. \quad (56)$$

Equations (25), (26), (27), (40), (41), (42), (43), (44), (45), (46), (48), (49), (50), (52), (55), and (56), along with proper transversality conditions characterize an equilibrium.

In what follows, a variable with a hat and time index denotes the percentage deviation of that variable from its steady state.

4.2.1 The steady state

Suppose $a = a^*$. We can show there is a symmetric steady state with $\beta = \beta^*$, $z = z^*$, and $s = \varepsilon = 1$. In this steady state, we have,

$$\frac{G}{y} = (1 - a) \tau,$$

$$r = \frac{1}{\beta} - 1 + \delta,$$

$$\frac{i}{z} = \frac{\alpha \delta}{\left[1 - \beta (1 - (1 - a) \tau)\right]}.$$

4.2.2 The log-linearized equilibrium system

The log-linearized versions of (25), (26), (27), (40), (42), (43), (44), (45), (46), (48), (49), (50), (52), (56) are

$$\dot{y} = a \dot{k} + (1 - a) \dot{t}, \quad (57)$$

$$\dot{w} = a (k - \dot{t}), \quad (58)$$
\[
\begin{align*}
\dot{r}_t &= (1 - \alpha)(\dot{h}_t - \dot{k}_t), \\
\dot{s}_t + \dot{\omega}_t + \dot{l}_t &= 0, \\
\dot{z}_t &= \left(1 - \frac{i}{z}\right)\dot{c}_t + \left(\frac{i}{z}\right)\dot{\omega}_t, \\
\dot{k}_{t+1} &= \hat{\delta}_t + (1 - \delta)\dot{k}_t, \\
-\sigma\dot{c}_t &= \hat{\phi}_t + \dot{s}_t - \dot{\varepsilon}_t, \\
\chi\dot{l}_t &= -\frac{\tau}{1 - \tau}\dot{r}_t + \dot{\phi}_t + \dot{w}_t, \\
\hat{q}_t &= \hat{\phi}_t + \dot{s}_t - \dot{\varepsilon}_t, \\
\hat{q}_t &= \epsilon_c\dot{c}_t + \epsilon_l\dot{l}_t + [1 - \beta(1 - \delta)](\varepsilon_{t+1} - \dot{s}_{t+1} + \dot{r}_{t+1}) + \dot{q}_{t+1}, \\
\varepsilon_{t+1} - \sigma\dot{c}_t &= \dot{e}_t - (\sigma + \epsilon_c)\dot{c}_t - \epsilon_l\dot{l}_t, \\
\varepsilon_t &= (1 - \alpha)\dot{s}_t, \\
\frac{1}{1 - (1 - \alpha)\tau}\hat{y}_t &= (1 - \alpha)(\hat{z}_t - \eta\varepsilon_t) + \eta\dot{s}_t,
\end{align*}
\]
characteristic roots are hard to obtain, so in what follows we focus on showing our results numerically. To fix parameter values, we set $\alpha = 0.33$, $\beta = 0.96$, $\delta = 0.1$, $\sigma = 1$, $\chi = 0$, $\epsilon_c = -0.5$, $\epsilon_l = 0$, as in SGU (1997, 2003), and $\eta = 0.5$, as in Corsetti et al. (2008).

Figure 4 displays the range of the steady-state labor income tax rate ($\tau$) for different degrees of openness ($\alpha$) of the small open economy under which (in)determinacy occurs.

As the figure shows, indeterminacy is less likely to occur, as the economy becomes more open to the rest of the world. Importantly, in light of the labor tax rates estimated by Mendoza et al. (1997) for developed small open economies, most OECD countries have a labor income tax rate less than 40%, with the OECD average at 33.459% (for the period 1965-1991): 25.36% for the US, 25.488% for the UK, 23.273% for Canada, 21.328% for Japan, 39.69% for France, and 36.832% for Germany. As can be seen from the figure, if labor income tax rate is less than 42%, then the economy has a determinantal equilibrium, regardless of the degree of its openness. The result is robust to alternative values of the model's parameters within their empirically plausible ranges – for instance, when the trade elasticity $\eta$, a key parameter that may matter for the result quantitatively, takes a higher value – further relieving possible worries about indeterminacy.

The above result shows the importance to extend our analysis to the new open economy macro model in order to explore the implications of endogenous international price adjustments that impinge on the core mechanism of indeterminacy. As we have shown above, in such a more general and more standard setting, the income effects that tend to make indeterminacy less likely to occur can be generated not only through cross-border capital movements due to international asset trade, as in the traditional small open economy model presented in the previous section, but more importantly, by endogenous terms-of-trade and other international price changes associated with international goods trade.

5 Conclusion

We have shown that, in contrast to the message from the existing closed economy studies, possible worries about self-fulfilling expectations and beliefs-driven aggregate instability associated with a balanced-budget fiscal policy rule with endogenous labor and capital income tax rates can be largely relieved for a small open economy because of the resultant income effects. According to our analysis in this paper, the income effects that make sunspot expectations unlikely to occur can be generated through cross-border capital movements arising from international asset trade as well as by endogenous terms-of-trade and other international price changes due to international goods trade. From a policy perspective, the results of this paper suggest that if the fiscal authority relies on changes in income tax rates to achieve budget balance then integrated world capital and goods markets can play a crucial role in stabilizing the economy.
The conclusion of this paper is based on local analysis. Stockman (2010) shows that global indeterminacy could still occur even in the presence of local determinacy in a closed economy model à la SGU (1997). It would be worthwhile to conduct global analysis to examine the robustness of the local results that we have obtained in an open economy setting in this paper. We leave this investigation to future research.

Appendix A: Equilibrium solution for the closed economy model

The first-order conditions for utility maximization are

\[ c_t^{-\sigma} = p_t^{-1} q_t, \]
\[ l_t^\bar{Y} = (1 - \tau_t) w_t p_t^{-1} q_t, \]
\[ \dot{q}_t = q_t (\rho + \delta - p_t^{-1} r_t). \]

The goods market clearing conditions are

\[ c_t = k_t^{\alpha_1} l_t^{1-\alpha_1} - T_{1t} = [1 - (1 - \alpha_1) \tau_t] \left( \frac{l_t}{k_t} \right)^{1-\alpha_1} \]
\[ p_t l_t = p_t k_t^{\alpha_2} l_t^{1-\alpha_2} - T_{2t} = [1 - (1 - \alpha_2) \tau_t] \left( \frac{l_t}{k_t} \right)^{1-\alpha_2} \]

If \( \alpha_1 \neq \alpha_2 \), the system reduces to the following two dynamic equations about \( q_t \) and \( k_t \),

\[ \dot{q}_t = \left[ \rho + \delta - \alpha_2 \nu^{1-\alpha_2} p(q_t, k_t) \right]^{\frac{1-\alpha_2}{\alpha_2-1}} q_t, \] \( A1 \)
\[ \dot{k}_t = \nu^{1-\alpha_2} \left[ 1 - (1 - \alpha_2) \tau (q_t, k_t) \right] p(q_t, k_t) \left( \frac{l_t}{k_t} \right)^{\frac{1-\alpha_2}{\alpha_2-1}} k_t (q_t, k_t) - \delta k_t. \] \( A2 \)

If \( \alpha_1 = \alpha_2 = \alpha \), then \( p_t = 1 \), and the system collapses to that in the one-sector closed economy model,

\[ \dot{q}_t = (\rho + \delta - \alpha k_t^{\alpha - 1} l_t^{1-\alpha}) q_t, \] \( A3 \)
\[ \dot{k}_t = k_t^{\alpha} l_t^{1-\alpha} - \delta k_t - c_t - G, \] \( A4 \)

where \( c_t, l_t, \) and \( \tau_t \) are functions of \( q_t \) and \( k_t \) solved from the following first-order conditions,

\[ c_t^{-\sigma} = q_t, \]
\[ l_t^\bar{Y} = (1 - \alpha)(1 - \tau_t) q_t k_t^{\alpha} l_t^{1-\alpha}, \]
\[ G = (1 - \alpha) \tau_t k_t^{\alpha} l_t^{1-\alpha}. \]
Appendix B: Proof of Corollary 1

We first notice that \( \tau^o \) must lie inside the interval \((\tau, 1)\) in order for the indeterminacy condition to hold. Since \( \Gamma(1) = -\frac{\rho\alpha}{(\sigma + \chi)(\rho + \delta)(1 - \alpha)} < 0 \) and \( \tau < 1 \), indeterminacy requires \( \Gamma(\tau) > 0 \), which immediately implies that there exists a lower bound \( \underline{\sigma} \) for \( \sigma \),

\[
\sigma > \underline{\sigma} \equiv \frac{(1 + \chi)\rho\alpha + (\rho + \delta)(1 - \alpha)^2}{\rho + \delta(1 - \alpha)} \frac{\alpha}{1 - \alpha} > 0,
\]

that is necessary for indeterminacy.

If \( \sigma < \underline{\sigma} \), then \( \Gamma(\tau) < 0 \) and \( \Gamma(0) > 0 \), and there exists a \( \tau^o \in (0, \tau) \). It then follows that indeterminacy can never occur:

(i) If the steady-state value \( \tau \in (\tau, 1) \) (which is a necessary condition for \( Tr(J) < 0 \)), then \( \Gamma(\tau) < 0 \) and \( D(J) < 0 \). The system is determinate.

(ii) If \( \tau \in (0, \tau) \) (i.e., \( \frac{1}{\tau + \delta} < \chi \)), then \( Tr(J) > 0 \). Either the system is determinate, if \( 0 < \tau < \tau^o \) (as \( D(J) < 0 \)), or there exist no equilibrium solutions, if \( \tau^o < \tau < \tau \) (as \( D(J) > 0 \)).

Thus, for \( \sigma < \underline{\sigma} \), the system is determinate if \( \tau \in (0, \tau^o) \cup (\tau, 1) \), and there are no equilibrium solutions if \( \tau \in (\tau^o, \tau) \).

Appendix C: Derivation of the 4 \times 4 system in 4.2.3

Using \( \hat{q}_t = -\sigma \dot{c}_t, \hat{e}_t = (1 - a) \dot{s}_t, \hat{\phi}_t = \chi \dot{h}_t + \frac{\tau}{\tau + \delta} \hat{c}_t - \dot{w}_t, \hat{y}_t = \alpha \dot{k}_t + (1 - \alpha) \hat{l}_t, \dot{w}_t = \alpha (\dot{k}_t - \dot{l}_t), \hat{r}_t = (1 - \alpha) (\dot{l}_t - \dot{k}_t) \), we can write the 13-equation system in 4.2.2 into a 6-equation system,

\[
\begin{align*}
\dot{z}_t &= \left(1 - \frac{i}{\tau} \right) \dot{c}_t + \left(\frac{i}{\tau} \right) \dot{e}_t, \\
\dot{k}_{t+1} &= \delta \dot{c}_t + (1 - \delta) \dot{k}_t, \\
-\sigma \dot{c}_t &= \left(\chi + \frac{\alpha - \tau}{1 - \tau} \right) \dot{c}_t - \left(\frac{\alpha}{1 - \tau} \right) \dot{k}_t + a \dot{\delta}_t, \\
\sigma \dot{c}_{t+1} - (\sigma + \epsilon_c) \dot{c}_t - \epsilon_l \dot{t}_t &= \left[1 - \beta (1 - \delta) \right] \left[-a \dot{s}_{t+1} + (1 - a) \left(\dot{l}_{t+1} - \dot{k}_{t+1} \right) \right], \\
\sigma \dot{c}_{t+1} - (\sigma + \epsilon_c) \dot{c}_t - \epsilon_l \dot{t}_t &= (1 - a) (\dot{s}_{t+1} - \dot{s}_t), \\
\frac{1}{1 - (1 - a) \tau} \left[\alpha \dot{k}_t + (1 - a) \dot{l}_t \right] &= (1 - a) \dot{z}_t + \eta a (2 - a) \dot{s}_t.
\end{align*}
\]

By introducing two auxiliary notations,

\[
\begin{align*}
\beta_\delta &\equiv 1 - \beta (1 - \delta), \\
\alpha_\tau &\equiv \frac{1}{1 - (1 - a) \tau},
\end{align*}
\]
which imply

\[ i_{z} \equiv \frac{i}{z} = \alpha \beta \alpha_{\tau} \beta_{\delta}^{-1}, \]

we can reduce the above 6-equation system, using \( \dot{z}_{t} = (1 - i_{z}) \hat{c}_{t} + i_{z} \hat{\delta}_{t} \), into a 5-equation system,

\[ \dot{\hat{k}}_{t+1} = \delta \hat{\delta}_{t} + (1 - \delta) \dot{\hat{k}}_{t}, \]

\[ -\sigma \hat{c}_{t} = \left( \chi + \frac{\alpha - \tau}{1 - \tau} \right) \hat{l}_{t} - \left( \frac{\alpha}{1 - \tau} \right) \hat{k}_{t} + a \hat{s}_{t}, \]

\[ \sigma \hat{c}_{t+1} - (\sigma + \epsilon_{c}) \hat{c}_{t} - \epsilon_{l} \hat{l}_{t} = \beta_{\delta} \left[ -a \hat{s}_{t+1} + (1 - \alpha)(\dot{\hat{l}}_{t+1} - \hat{k}_{t+1}) \right], \]

\[ \sigma \hat{c}_{t+1} - (\sigma + \epsilon_{c}) \hat{c}_{t} - \epsilon_{l} \hat{l}_{t} = (1 - a)(\dot{\hat{s}}_{t+1} - \hat{s}_{t}), \]

\[ \alpha_{\tau} \left[ \alpha \hat{k}_{t} + (1 - \alpha) \hat{\delta}_{t} \right] = (1 - a) \left[ (1 - i_{z}) \hat{c}_{t} + i_{z} \hat{\delta}_{t} \right] + \eta a (2 - a) \hat{s}_{t}. \]

By introducing three more auxiliary notations,

\[ c_{s} \equiv \frac{a}{\sigma}, \]

\[ a_{l} \equiv -\frac{\chi}{\sigma} - \frac{\alpha - \tau}{\sigma(1 - \tau)}, \]

\[ c_{k} \equiv \frac{\alpha}{\sigma(1 - \tau)}, \]

we can further simplify the above 5-equation system, using \( \hat{c}_{t} = c_{s} \hat{s}_{t} + c_{l} \hat{l}_{t} + c_{k} \hat{k}_{t} \), and \( \hat{l}_{t} = \frac{1}{\delta} \dot{\hat{k}}_{t+1} - \frac{1 - \delta}{\delta} \dot{\hat{k}}_{t} \), into a 3-equation system,

\[ (1 - a + a \beta_{\delta}) \dot{\hat{s}}_{t+1} - \beta_{\delta}(1 - a) \hat{l}_{t+1} + \beta_{\delta}(1 - a) \hat{k}_{t+1} = (1 - a) \hat{s}_{t}, \]

\[ (\sigma c_{s} - 1 + a) \dot{\hat{s}}_{t+1} + \sigma c_{q} \hat{l}_{t+1} + \sigma c_{k} \hat{k}_{t+1} = [(\sigma + \epsilon_{c}) c_{s} - 1 + a] \dot{\hat{s}}_{t} + [(\sigma + \epsilon_{c}) c_{l} + (1 - \alpha) \hat{c}_{t} + (\sigma + \epsilon_{c}) c_{k} \hat{k}_{t}, \]

\[ -\frac{(1 - a) i_{z}}{\delta} \dot{\hat{k}}_{t+1} = [(1 - a)(1 - i_{z}) c_{s} + \eta a (2 - a)] \dot{\hat{s}}_{t} + [(1 - a)(1 - i_{z}) c_{l} - \alpha_{\tau}(1 - a)] \dot{\hat{l}}_{t} \]

\[ + \left[ (1 - a)(1 - i_{z}) c_{k} - \alpha_{\tau} \alpha - \frac{(1 - \delta)(1 - a) i_{z}}{\delta} \right] \dot{\hat{k}}_{t}. \]

Finally, we can use (41) and (49) to obtain

\[ \beta_{\delta} \dot{\hat{b}}_{t+1} - \dot{\hat{b}}_{t} = \dot{\hat{y}}_{t} - \alpha_{\tau}^{-1}(\dot{\hat{s}}_{t} - \dot{\hat{c}}_{t} + \dot{\hat{\delta}}_{t}), \]

which implies

\[ \frac{i_{z}}{\alpha_{\tau} \delta} \dot{\hat{k}}_{t+1} + \beta_{\delta} \dot{\hat{b}}_{t+1} = \]

\[ - \left[ a + (1 - i_{z}) c_{s} \right] \dot{\hat{s}}_{t} + \left[ 1 - \alpha - \frac{(1 - i_{z}) c_{l}}{\alpha_{\tau}} \right] \dot{\hat{l}}_{t} + \left[ \alpha - \frac{(1 - i_{z}) c_{k}}{\alpha_{\tau}} + \frac{(1 - \delta) i_{z}}{\alpha_{\tau} \delta} \right] \dot{\hat{k}}_{t} + \dot{\hat{b}}_{t}. \]

These four equations form the 4 × 4 system in 4.2.3, where we have introduced another auxiliary notation

\[ a_{z} \equiv (1 - a)(1 - i_{z}). \]
References


Figure 1. The closed economy model: D - determinacy, I - indeterminacy, N - no equilibrium
Figure 2. The traditional small open economy model with imperfect international capital mobility under the balanced-budget labor income tax rule: D - determinacy, I - indeterminacy, N - no equilibrium
Figure 3. The traditional small open economy model with imperfect international capital mobility under the balanced-budget labor and capital income tax rule: D - determinacy, I - indeterminacy
Figure 4. The new small open economy macro model: D - determinacy, I - indeterminacy