Balanced-budget rules and aggregate instability: the role of capital utilization

Kevin X. D. Huang\textsuperscript{a}, Qinglai Meng\textsuperscript{b}

\textsuperscript{a}Department of Economics, Vanderbilt University, Nashville, TN 37235-1819, USA
\textsuperscript{b}Department of Economics, Oregon State University, Corvallis, OR 97331, USA

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Abstract

Schmitt-Grohé and Uribe (1997) demonstrate that a balanced-budget fiscal policy can induce aggregate instability unrelated to economic fundamentals. The empirical relevance of this result has since been challenged by several subsequent studies. In this paper we show that such extrinsic instability is an empirically robust plausibility associated with a balanced-budget rule once endogenous capital utilization is taken into consideration. This suggests that the consideration, design, or operation of a balanced-budget fiscal policy must recognize that it may constitute a practical source of self-fulfilling prophecies and belief-driven fluctuations.

\textit{JEL classification:} E32; E62

\textit{Keywords:} Balanced-budget rules; Aggregate instability; Capital utilization
1 Introduction

A recurrent debate in academic and policymaking circles is whether a government should operate under a balanced budget. On the one side, such fiscal discipline is considered a necessary tool to restrict deficit spending and limit government debt increment in order to ensure fiscal sustainability and long-run growth. On the other side, it is excoriated as a constraint on government’s ability to deal with fundamental shocks, especially large adverse shocks such as wars and natural disasters, resulting in amplified, rather than dampened, short-run fluctuations. The benefit-cost comparison, along with certain operational considerations, has held a center stage in the debate surrounding a balanced-budget rule.\(^1\) A clear understanding of the associated costs against its benefits is of critical importance in the consideration, design, or operation of a balanced-budget fiscal policy.\(^2\)

The objective of this paper is to emphasize a cost associated with a balanced-budget rule which is not as publicized as the one highlighted above, but which may be more challenging to cope with, as it takes the form of extrinsic instability unrelated to economic fundamentals. The point that a balanced-budget rule, under which a government collects enough revenues by taxing labor and capital incomes to finance its expenditures, can be destabilizing, even in the absence of fundamental shocks, was first demonstrated by Schmitt-Grohé and Uribe (1997, SGU henceforth) using a canonical neoclassical model. While this result has spurred a series of works,\(^3\) its empirical relevance is challenged by several subsequent studies showing that such extrinsic instability may not arise if consumption taxes are included in the government’s revenues (e.g., Giannitsarou 2007), or if the government has a large or even just a moderate debt outstanding (e.g., Stockman 1998). In particular, for the several countries examined by SGU, once their consumption tax rates or their government debt-to-GDP ratios are taken into account, these studies show, the implementation or operation of the balanced-budget rules in these countries does not induce any extrinsic instability.

\(^1\)See Azzimonti (2013) for a recent survey of the debate and related literature.
\(^2\)In practice, several European countries and all U.S. states other than Vermont have some forms of balanced budget provisions in their constitutions or basic laws, while there have also been repeated attempts to add a balanced-budget rule to the national United States Constitution.
originally concerned by SGU when these real-world features were abstracted away.

It is a message of this paper that the issue raised by SGU can be empirically relevant, even with the aforementioned real-world features taken into account, once another real-world feature is factored into consideration. We show that in this more realistic setting a balanced-budget rule is much more likely to induce extrinsic instability and it does so in an empirically plausible and robust manner. In particular, for those countries examined by SGU, fluctuations in economic activities can emerge under a balanced-budget rule, not only at their current but much higher consumption tax rates and government debt-to-GDP ratios, even in the absence of fundamental shocks. This is to say that, the consideration or operation of a balanced-budget rule in these countries must recognize that it may constitute a practical source of self-fulfilling prophecies and belief-driven fluctuations.\(^4\)

The real-world feature that is incorporated into our analysis is endogenous capital utilization. The concept of capital utilization as an optimal decision dates back to Keynes (1936), which has been further developed by Taubman and Wilkinson (1970) and others. The essential ingredient is that increasing capital utilization increases the user cost of capital through an acceleration of capital depreciation. As a consequence, firms will not, in general, find it optimal to fully utilize the stock of capital, preferring to “hoard” some capital instead so that they can use it more intensively when the returns to doing so are unusually large. This phenomenon is in line with the evidence documented in many empirical studies.\(^5\)

Respecting this realistic feature has proven to be important for deciphering a number of puzzles concerning the business cycle and growth.\(^6\) What is relevant for the result in this paper is the fact that endogenous capital utilization effectively increases equilibrium output-labor elasticity and decreases equilibrium output-capital elasticity. This redistribution of equilibrium factor elasticities makes it much easier for sunspot expectations to be self-fulfilling. This explains why endogenous capital utilization plays a decisive role for the paper’s result.

The rest of the paper is organized as follows. The next section sets up the model by introducing

\(^4\)The literature has identified other potential sources of self-fulfilling expectations. There has also been an increased recognition of the practical relevance of belief-driven instability unrelated to economic fundamentals. For instance, sunspot expectations and belief coordination failures as a potential source of the recent financial crisis and ensuing recession is stressed by Farmer (2010) and also acknowledged by Lucas and Stokey (2011).

\(^5\)See Chatterjee (2005) for a recent survey.

capital utilization as an optimal decision to the model in SGU (1997). Their balanced-budget rule is also generalized by including consumption tax as part of the government revenues, besides the tax revenues on labor and capital incomes. Section 4 investigates the stability properties of equilibrium. In contrast to Giannitsarou (2007), it is shown in this section that aggregate instability induced by the fiscal policy arises well within the range of the tax rates for the United States and other industrialized countries. Section 4 extends the analysis and assesses the quantitative implications of the presence of public debt for the policy-induced aggregate instability. It is shown again that the results in Section 3 are robust to this realistic extension. Section 5 concludes.

2 The model

Our model economy consists of households, firms and the government.

Households

The economy is populated by a unit measure of identical infinitely lived households. The representative household maximizes the present discounted value of its lifetime utility

$$\max_{\alpha} \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \eta \frac{L_t^{1+\chi}}{1+\chi} \right), \quad \beta \in (0, 1), \chi \geq 0, \eta > 0$$

(1)

where $\beta$ is the discount factor, $C_t$ consumption, $L_t$ hours worked, and $\chi$ denotes the inverse of the intertemporal elasticity of substitution for labor supply. Households derive income from providing labor and capital services to firms. The budget constraint faced by the representative household is

$$C_t + I_t + T_t = w_t L_t + r_t u_t K_t, \quad K_0 > 0 \text{ given}$$

(2)

where $K_t$ is the household’s capital stock, $I_t$ investment, $T_t$ the total taxes, $w_t$ the real wage rate, $r_t$ the rental rate for capital services. $u_t \in [0, 1]$ represents the rate of capital utilization (i.e., the number of hours per period or the speed per hour at which the capital stock is operated). For a given $K_t$, $u_t$ determines the flow of capital services, $u_t K_t$. The law of motion for the capital stock is

$$K_{t+1} = (1 - \delta_t) K_t + I_t$$

(3)
where $\delta_t$ is the rate of capital depreciation defined as function of the capital utilization rate

$$\delta_t = \frac{1}{\theta} u_t^\theta, \quad \theta > 1$$

(4)

The capital accumulation equation is standard except for the variable depreciation rate. Depreciation $\delta_t$ is an increasing convex function of $u_t$, as first formulated in Taubman and Wilkinson (1970) and Greenwood, et al. (1988). This depreciation function captures Keynes’s notion of the user cost to capital - higher utilization causes faster depreciation, at an increasing rate, because of wear and tear on the capital stock.

**Firms**

There is a continuum of identical competitive firms in the economy, with the total number normalized to one. Each firm produces output $Y_t$ with a constant-returns-to-scale Cobb-Douglas technology, rents capital at a rate $r_t$ and hires labor at a rate $w_t$, and maximizes profits in every period

$$\max(Y_t - r_t(u_t K_t) - w_t L_t)$$

s.t. $Y_t = A(u_t K_t)^\alpha L_t^{1-\alpha}, \quad \alpha \in (0, 1), \quad A > 0$ (6)

Under the assumption that factor markets are perfectly competitive, the first-order conditions of the firm are given by

$$r_t = A\alpha (u_t K_t)^{\alpha-1} L_t^{1-\alpha} = \alpha \frac{Y_t}{u_t K_t}$$

(7)

$$w_t = A(1 - \alpha) (u_t K_t)^\alpha L_t^{-\alpha} = (1 - \alpha) \frac{Y_t}{L_t}$$

(8)

**Government**

The government revenue includes labor and capital income taxes, as well as consumption taxes. The government faces an exogenous stream of constant expenditure $G$, and the initial stock of public debt is zero. The government is subject to a balanced-budget requirement, and its budget constraint is then given by

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7 As the firm cares about capital utilization, the first-order condition with respect to either $u_t$ or $K_t$ alone, or $(u_t K_t)$ as a whole would yield the same equation (7).
\[ G = T_t = \tau^l_t w_t L_t + \tau^k_t (r_t u_t - \delta_t) K_t + \tau^n_t C_t \]  

(9)

where \( \tau^l_t \) and \( \tau^k_t \) denote the labor and capital income tax rates, and \( \tau^n_t \) is the consumption tax rate. As in SGU (1997), the term \( -\tau^k_t \delta_t K_t \) represents a depreciation allowance. Here the tax rates are allowed to adjust endogenously to balance the budget.\(^8\)

3 Dynamics and aggregate (in)stability

The household’s budget constraint (2), combined with equation (3) and the government’s budget constraint (9), can be rewritten as

\[ K_{t+1} = K_t + \left(1 - \tau^l_t\right) w_t L_t + \left(1 - \tau^k_t\right) (r_t u_t - \delta_t) K_t - (1 + \tau^n_t) C_t \]  

(10)

The first-order conditions for the household’s optimization problem are given by (denoting \( \Lambda_t \) as the costate variable)

\[ \Lambda_t = \beta \left[ \Lambda_{t+1} \left(1 + \left(1 - \tau^k_{t+1}\right) (r_{t+1} u_{t+1} - \delta_{t+1})\right)\right] \]  

(11)

\[ (1 + \tau^n_t) \Lambda_t = C_t^{-\sigma} \]  

(12)

\[ \left(1 - \tau^l_t\right) w_t \Lambda_t = \eta L_t^\chi \]  

(13)

\[ r_t = u_t^{\theta-1} \]  

(14)

together with the aggregate resource constraint for the economy

\[ C_t + G + K_{t+1} - K_t + \delta_t K_t = A (u_t K_t)^\alpha L_t^{1-\alpha} \]  

(15)

Notice that equation (14) determines the optimal rate of capital utilization by equating its marginal benefit to marginal cost. The LHS represents the marginal return gained by increasing the capital utilization rate, and the RHS is the marginal loss in terms of capital depreciation due to the intensified usage of existing capital stock. Furthermore, as the marginal returns by increasing

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\(^8\)As in SGU (1997), it is assumed here that all government expenditures consist of purchases of goods. Our results are not affected if instead all tax revenues are returned to the public in the form of lump-sum tax revenues, or if we allow for productive or utility-generating government purchases. Furthermore, the results in this paper are robust to the case of income-elastic government expenditures.
the capital utilization is equal to the marginal product of capital, it implies that capital should be used more intensively during economic booms when its marginal product is high and less intensively during recessions when its marginal product is low.

Consider a log-linear approximation of the equilibrium conditions (7), (8), and (11)-(15). Let \( \lambda_t, k_t, l_t, \tau^j_t \) \((j = l, k, c)\) denote the log deviations of \( \Lambda_t, K_t, L_t, \) and \( \tau^j_t \) \((j = l, k, c)\) from their respective steady states. In the appendix, we reduce the log-linearized system to the following two equations:

\[
\begin{pmatrix}
  k_{t+1} \\
  \lambda_{t+1}
\end{pmatrix} = \begin{pmatrix}
  J \\
  0
\end{pmatrix}
\begin{pmatrix}
  k_t \\
  \lambda_t
\end{pmatrix}
\]  

(16)

where \( J \) is the two by two Jacobian matrix of this linear system. Since (16) contains only one predetermined variable, \( k_t \), the system exhibits instability if and only if both eigenvalues of \( J \) are less than 1 in modulus. In such a case, persistent and recurring fluctuations in aggregate activity become possible even in the absence of shocks to fundamentals. This kind of self-fulfilling fluctuations are also welfare-reducing. On the other hand, the system is saddle-path stable if one eigenvalue lies outside and the other inside the unit circle.

As the system cannot be analytically solved, we calibrate the model using a baseline parameterization that reflects typical values found in the real business cycle literature. We set \( \chi = 0 \) so that the model is in line with the model of indivisible labor of Hansen (1985) and the benchmark analysis of SGU (1997). The time unit is taken to be a quarter of a year. The actual capital-output elasticity (equal to the capital share), \( \alpha \), is set equal to 0.3, and the discount factor, \( \beta \), is chosen to be 0.99, and the steady-state value of capital depreciation, \( \delta \), is 0.025. The elasticity \( \theta \) can be derived from the steady-state conditions, i.e., \( \theta = \frac{1/\beta - 1 + \delta}{\delta} = 1.4 \) (which is the value calibrated by Greenwood et al., 1988).

### 3.1 The case without consumption taxes

To facilitate the comparison we illustrate the results of both the present paper and SGU (1997) in the same figure, Fig. 1, on the effects of a balanced-budget rule in the absence of consumption taxes. The labor and capital income tax rates are based on the values estimated by Mendoza et al. (1994) for the United States (US), the United Kingdom (UK), Germany (GE), Canada (CA), and
Japan (JP).

All the five countries fall into the instability region in both models. However, it is apparent the instability region is substantially larger than that in SGU (1997). As will be clear, the smaller instability region in SGU (1997) makes their results fragile, and their results may fail to hold when taking into account other realistic factors in the economy. By contrast, in our model with endogenous capital utilization, the instability results are robust to these considerations.

**Intuition**

SGU (1997) identify a necessary condition for instability: for the policy-induced instability to occur, the labor tax rate must be greater than a lower bound, which is approximately equal to the capital-output elasticity. If the utilization rate and the depreciation rate are constant as in SGU (1997), for the same Cobb-Douglas production function in (6), this elasticity would be $\alpha$, which is about 0.3 for the US. However, in the present model, from equations (6), (7), and (14), the optimal rate of capital utilization and the reduced-form aggregate production function evaluated are given by

$$u_t = (\alpha A \frac{L_t}{K_t})^\frac{1-\alpha}{\beta-\alpha}$$  \hspace{1cm} (17)

$$Y_t = (A\alpha)^\frac{\alpha}{\beta-\alpha} (K_t)^{\alpha(\frac{\theta-1}{\beta-\alpha})} L_t^{(1-\alpha)}\frac{\theta}{\beta-\alpha}$$  \hspace{1cm} (18)

From equation (17), the optimal rate of capital utilization is an increasing function of labor but a decreasing function of the capital stock. Thus, and as can be seen from equation (18), capital utilization effectively alters the equilibrium production function and has significant effects on the distribution of factor elasticities - the capital elasticity decreases and the labor elasticity increases (because $\frac{\theta-1}{\beta-\alpha} < 1$ and $\frac{\theta}{\beta-\alpha} > 1$). For example, for $\alpha = 0.3$ and $\theta = 1.4$, then the effective capital-output elasticity is just about 0.1, while the actual capital-output elasticity is 0.3, and the effective labor-output elasticity is around 0.9 while its actual value is 0.7. Therefore, the lower bound of the labor income tax rate for instability to arise under the balanced-budget rule is effectively smaller, and the instability region becomes much larger.

For the case with only labor income taxes, SGU (1997) analytically show that $\tau^l > \alpha$ is a necessary condition for instability to arise. This condition implies that the “equilibrium labor demand schedule” is upward sloping and steeper than the labor supply schedule. In the present model, this condition becomes $\tau^l > \alpha\left(\frac{\theta-1}{\beta-\alpha}\right)$, which is significantly weaker.
3.2 The case with income and consumption taxes

Giannitsarou (2007) studies the (in)stability issue by combining income taxes and consumption taxes in the balanced-budget fiscal policy in the model of SGU (1997). She shows that by taking into account the consumption taxes the possibility of the policy-induced aggregate instability is reduced, and hence consumption taxes have a stabilizing effect. We can illustrate her result in Fig. 2, which shows that the instability region shrinks as the consumption rates increases. In particular, as is also duplicated in Tables 1 and 2, for the same five countries considered, she finds that with consumption taxes three of them (the US, the UK, and Japan) become stable. In contrast, we find that with endogenous capital utilization the instability region is not affected much by including the consumption taxes, as is illustrated in Fig. 3. As can be seen from Table 2, all the five countries are unstable under the balanced-budget policy rule which includes both income and consumption taxes.

3.3 Capital versus consumption taxes

Because of the stabilizing effects of the consumption taxes, Giannitsarou (2007) presents a new argument in favor of consumption taxes in place of capital taxes. To support her argument, for the same five countries with income taxes, for each country she replaces capital income taxes by consumption taxes while holding the expenditures fixed. There is an implied consumption tax rate for each country (e.g., 10.87% for the US). Then it is found that four countries become stable with this new balanced-budget fiscal policy. Again, by contrast, with endogenous capital utilization, all the five countries fall into the instability region under the same policy proposed by Giannitsarou (2007). Table 3 presents these results (\(\bar{\tau}^c\) denotes the implied consumption tax rates if the capital taxes are replaced by consumption taxes).

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\(^9\) The fact that capital taxation creates large distortions has been documented extensively in the literature. In terms of welfare, it has been shown that the loss associated with capital taxes is much larger than the loss associated with consumption taxes (Cooley and Hansen, 1992).
4 Public debt and aggregate (in)stability

As in SGU (1997), it is assumed in Section 2 that the initial public debt is zero. Stockman (1998) finds, however, that such an assumption is not innocuous. In particular, Stockman (1998) demonstrates that the higher the debt to GDP ratio, the less likely it is for the economy to be destabilized by the balanced-budget fiscal policy rules. Indeed, with the same income tax rates as those used in SGU (1997) and with the available data in the early 1990s on the debt to GDP ratio for the US, the UK, Canada, and Germany, with the exception of Germany, the other three countries would not be destabilized by the balanced-budget policy rule. Given that it is typical that a long period of sustained-deficit spending and a high debt to GDP ratio is a usual antecedent to a balanced-budget debate, he concludes that the conditions are favorable for the adoption of such a policy to result in economic stability.

This section reexamines the issue in the framework with endogenous capital utilization. In this case, the balanced-budget rule (9) becomes

\[ G + \left(1 - \tau_k^k\right) R_t B = \tau^k w_t L_t + \tau^k (r_t u_t - \delta_t) K_t + \tau^C C_t \] (19)

where the new term on the left side, \(1 - \tau_k^k\) \(R_t B\) denotes after-tax interest payments on the public debt; \(B > 0\) denotes the stock of debt, which is constant by the balanced-budget rule; and \(R_t\) denotes the rate of interest paid on the debt, which in equilibrium must be equal to \(r_t u_t - \delta_t\).

Figure 4 shows the results in SGU (1997) and Stockman (1998) under the balanced-budget rule without consumption taxes. The instability region decreases as the debt to GDP ratio increases. If the debt to GDP ratio is zero, then this collapses to the case in SGU (1997) without zero government debt, and all the five countries considered above are unstable. However, if the debt to GDP ratio is above 50%, then the US, the UK, Canada and Japan will move to the stability region.

In stark contrast, with endogenous capital utilization, as is shown in Fig. 5, for the empirically plausible values of the debt to GDP ratio, the instability results are not affected (even for the debt to GDP ratio of 450%, all the five countries remain in the instability region).\(^{10}\)

\(^{10}\)Here we consider the balanced-budget rule which does not include the consumption tax as it plays an insignificant role. Adding the consumption tax does not affect the results at all.
5 Concluding remarks

We have shown that expectation-driven fluctuations unrelated to economic fundamentals can be a practical issue associated with a balanced-budget rule. Such belief-induced, extrinsic instability is an empirically robust plausibility once endogenous capital utilization is taken into consideration. This poses a unique challenge to governments in their consideration or operation of balanced-budget fiscal policies.

Previous studies suggest that monetary policies can be an especially effective tool to preempt sunspot expectations and stabilize belief-driven fluctuations. This suggests that one promising approach in meeting the above challenge may require coherent designs of and proper interactions between fiscal and monetary policies. We leave this topic for future research.
6 Appendix

By substitutions the system can be summarized by the following equations

\[ \Lambda_t = \beta \left\{ \Lambda_{t+1} \left[ 1 + \left( 1 - \tau^k_{t+1} \right) \left( \alpha \frac{Y_{t+1}}{K_{t+1}} - \frac{1}{\theta} u^\theta_{t+1} \right) \right] \right\} \]

\[ K_{t+1} = Y_t + \left( 1 - \frac{1}{\theta} u^\theta_t \right) K_t - C_t - G \]

\[ \eta L_t^k = \left( 1 - \tau^l_t \right) (1 - \alpha) \frac{Y_t}{L_t} \Lambda_t \]

\[ u^\theta_t = \alpha \frac{Y_t}{K_t} \]

\[ (1 + \tau^c_t) \Lambda_t = C_t^{-\sigma} \]

\[ Y_t = A \left( u_t K_t \right)^{\alpha} L_t^{1-\alpha} \]

\[ G = \tau^l_t (1 - \alpha) Y_t \frac{\theta - 1}{\theta} - \tau^k_t \alpha Y_t + \tau^c_t C_t \]

The steady-state values \((C, K, L, \Lambda, Y)\) are determined by the following system of equations:

\[ \frac{Y}{K} = \left[ \frac{1 - \beta}{(1 - \tau^{c}) \beta} + \delta \right] \frac{1}{\alpha} \]

\[ \frac{1 - \beta}{(1 - \tau^{k}) \beta \delta} + 1 = \theta \]

\[ \theta = 1 + \frac{1 - \beta}{\beta \delta (1 - \tau^{k})} \]

\[ \delta K = AK^{\alpha} L^{1-\alpha} - C - G \]

Thus, we have

\[ \frac{C}{K} = \frac{\left[ 1 - \tau^l (1 - \alpha) - \tau^k \alpha \frac{\theta - 1}{\theta} \right] Y}{1 + \tau^c} \]

Here we use \(\tau^j\) to denote the steady state values for \(\tau^j_t\) \((j = l, k, c)\). Log-linearizing the system yields the following equations

\[ -\frac{\tau^l}{1 - \tau^l} \tau^l + y_t - l_t + \lambda_t = \chi l_t \]

\[ \theta \tilde{u}_t = y_t - k_t \]

\[ \lambda_t = -\sigma c_t - \frac{\tau^c}{1 + \tau^c} \tau^c_t \]
\[ y_t = \alpha (\bar{u}_t + k_t) + (1 - \alpha) l_t \]

\[ 0 = (1 + e) \tilde{\tau}_t + y_t + \epsilon e_t \]

where \( e = \frac{\tau^c}{\tau^l (1 - \alpha) + \tau^k (\frac{\tau^c}{\tau^l})} \). Here it is assumed that \( \tau^l \), \( \tau^k \), and \( \tau^c \) vary in the same proportion to balance the budget, i.e., \( \tilde{\tau}_t^l = \tilde{\tau}_t^k = \tilde{\tau}_t^c = \tilde{\tau}_t \). In addition, with some substitutions, we obtain

\[ l_t = \frac{1 - \epsilon_2 \frac{\tau^l}{\sigma}}{1 + \chi - \epsilon_1 \frac{(1 - \alpha)\theta}{\gamma - \alpha}} \lambda_t + \frac{\epsilon_1 \frac{\alpha \theta - 1}{\gamma - \alpha}}{1 + \chi - \epsilon_1 \frac{(1 - \alpha)\theta}{\gamma - \alpha}} k_t = l_\lambda \lambda_t + l_k k_t \]

where \( \epsilon_1 = 1 + \frac{\tau^l}{1 + e - \frac{\tau^l}{\sigma}} \), \( \epsilon_2 = \frac{\tau^l}{1 + e - \frac{\tau^l}{\sigma}} \), and

\[ \tilde{\tau}_t = \frac{\epsilon - \theta \frac{(1 - \alpha)}{\gamma - \alpha} l_\lambda}{1 + \epsilon - \frac{\tau^c}{\sigma}} \lambda_t - \frac{\alpha \frac{\theta - 1}{\gamma - \alpha} + \theta \frac{(1 - \alpha)}{\gamma - \alpha} l_k}{1 + \epsilon - \frac{\tau^c}{\sigma}} k_t = \tau_\lambda \lambda_t + \tau_k k_t \]

\[ y_t = \frac{(1 - \alpha) \theta}{\beta - \alpha} l_\lambda \lambda_t + \left[ \frac{\theta - 1}{\beta - \alpha} + (1 - \alpha) \frac{\theta}{\beta - \alpha} l_k \right] k_t = y_\lambda \lambda_t + y_k k_t \]

The log-linearized system can be reduced to the following two equations

\[
\begin{pmatrix}
  k_{t+1} \\
  \lambda_{t+1}
\end{pmatrix} = \begin{pmatrix}
  1 & 0 \\
  b_k & b_c
\end{pmatrix}^{-1} \begin{pmatrix}
  d_k & d_c \\
  0 & 1
\end{pmatrix} \begin{pmatrix}
  k_t \\
  \lambda_t
\end{pmatrix} \equiv \mathbf{J} \begin{pmatrix}
  k_t \\
  \lambda_t
\end{pmatrix}
\]

where

\[ d_k = 1 + \frac{(\theta - \alpha) Y y_k}{\theta K} + \frac{C}{\sigma K} \frac{\tau^c \tau_k}{1 + \tau^c} \]

\[ d_c = \frac{(\theta - \alpha) Y y_k}{\theta K} + \frac{C}{\sigma K} \left( 1 + \frac{\tau^c}{1 + \tau^c} \tau_\lambda \right) \]

\[ b_k = \frac{\alpha \beta (\theta - 1) (1 - \tau^k) Y}{\theta} \left( y_k - 1 - \frac{\tau^k}{1 - \tau^k} \tau_k \right) \]

\[ b_c = 1 + \frac{\alpha \beta (\theta - 1) (1 - \tau^k) Y}{\theta K} \left( y_\lambda - \frac{\tau^k}{1 - \tau^k} \tau_\lambda \right) \]
References


Fig. 1. Aggregate (in)stability: SGU (1997) vs. our model. S - stability region, I - instability region
Fig. 2. Consumption tax and aggregate (in)stability in Giannitsarou (2007)
Fig. 3. Consumption tax and aggregate (in)stability with endogenous capital utilization
Fig. 4. Debt-GDP ratio and aggregate (in)stability in SGU (1997) and Stockman (1998)
Fig. 5. Debt-GDP ratio and aggregate (in)stability with endogenous capital utilization
Table 1
Estimated Tax Rates, 1996

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*Updated estimates from Mandoza et al. (1994); available online from the authors.

Table 2
Stability (Instability) With and Without Consumption Taxes

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<td>$\tau^c \neq 0$</td>
<td>$\tau^c = 0$</td>
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Table 3
Substituting Capital with Consumption taxes

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